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A RESULT ON (g, f, n)-CRITICAL GRAPHS*

Sizhong Zhou

Abstract

Let G be a graph, and let g, f be two integer-valued functions defined on $V(G)$ with $0 \le g(x) \le f(x)$ for each $x \in V(G)$. Then a spanning subgraph F of G is called a (g, f)-factor if $g(x) \le d_F(x) \le f(x)$ holds for each $x \in V(G)$. A graph G is said to be (g, f, n)-critical if $G - N$ has a (g, f)-factor for each $N \subseteq V(G)$ with $|N| = n$. In this paper, we obtain a neighborhood condition for a graph G to be a (g, f, n)-critical graph. Furthermore, it is shown that the result in this paper is best possible in some sense.

1 Introduction

All graphs considered in this paper will be finite and undirected simple graphs. Let G be a graph. We denote by $V(G)$ and $E(G)$ the set of vertices and the set of edges, respectively. For $x \in V(G)$, the degree of x and the set of vertices adjacent to x in G are denoted by $d_G(x)$ and $N_G(x)$, respectively. The minimum vertex degree of G is denoted by $\delta(G)$. For $S \subseteq V(G)$, the neighborhood of S is defined as:

$$N_G(S) = \bigcup_{x \in S} N_G(x).$$

For $S \subseteq V(G)$, we denote by $G[S]$ the subgraph of G induced by S, and $G - S = G[V(G) \setminus S]$. A vertex set $S \subseteq V(G)$ is called independent if $G[S]$

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has no edges. Let r be a real number. Recall that $\lfloor r \rfloor$ is the greatest integer such that $\lfloor r \rfloor \leq r$.

Let g, f be two integer-valued functions defined on $V(G)$ with $0 \leq g(x) \leq f(x)$ for each $x \in V(G)$. Then a spanning subgraph F of G is called a (g, f)-factor if $g(x) \leq d_F(x) \leq f(x)$ holds for each $x \in V(G)$. Let a and b be two integers with $0 \leq a \leq b$. If $g(x) = a$ and $f(x) = b$ for each $x \in V(G)$, then a (g, f)-factor is called an $[a, b]$-factor. A graph G is said to be (g, f, n)-critical if $G - N$ has a (g, f)-factor for each $N \subseteq V(G)$ with $|N| = n$. If $g(x) = a$ and $f(x) = b$ for each $x \in V(G)$, then a (g, f, n)-critical graph is called an (a, b, n)-critical graph. If $a = b = k$, then an (a, b, n)-critical graph is simply called a (k, n)-critical graph. The other terminologies and notations not given in this paper can be found in [1].

Liu and Yu [2] studied the characterization of (k, n)-critical graphs. Enomoto et al [3] gave some sufficient conditions of (k, n)-critical graphs. The characterization of (a, b, n)-critical graph with $a < b$ was given by Liu and Wang [4]. Zhou [5–7] gave some sufficient conditions for graphs to be (a, b, n)-critical. Li [8,9] gave some sufficient conditions for graphs to be (a, b, n)-critical graphs. A necessary and sufficient condition for a graph to be (g, f, n)-critical was given by Li and Matsuda [10]. Zhou [11–13] obtained some sufficient conditions for graphs to be (g, f, n)-critical graphs. Liu [14] found a binding number and minimum degree condition for a graph to be (g, f, n)-critical.

The following result was obtained by Berge and Las Vergnas [16], and by Amahashi and Kano [15], independently.

Theorem 1. *Let $b \geq 2$ be an integer. Then a graph G has an $[1, b]$-factor if and only if*

$$|N_G(S)| \geq \frac{|S|}{b},$$

for all independent subsets S of $V(G)$.

In [17], Kano showed the following result on neighborhood conditions for the existence of $[a, b]$-factors.

Theorem 2. *Let a and b be integers such that $2 \leq a < b$, and let G be a graph of order p with $p \geq 6a + b$. Suppose, for any subset $X \subset V(G)$, we have*

$$N_G(X) = V(G), \qquad if \qquad |X| \geq \left\lfloor \frac{bp}{a+b-1} \right\rfloor$$

or

$$|N_G(X)| \geq \frac{a+b-1}{b}|X|, \qquad if \qquad |X| < \left\lfloor \frac{bp}{a+b-1} \right\rfloor.$$

Then G has an $[a, b]$-factor.

Zhou [5] obtained the following result on neighborhoods of independent sets for graphs to (a,b,n)-critical graphs.

Theorem 3. *Let a, b and n be nonnegative integers with $1 \le a < b$, and let G be a graph of order p with $p \ge \frac{(a+b)(a+b-2)}{b} + n$. Suppose that*

$$|N_G(X)| > \frac{(a-1)p + |X| + bn - 1}{a+b-1},$$

for every non-empty independent subset X of $V(G)$, and

$$\delta(G) > \frac{(a-1)p + a + b + bn - 2}{a+b-1}.$$

Then G is an (a,b,n)-critical graph.

Zhou [11] gave a binding number condition for a graph to be a (g,f,n)-critical graph.

Theorem 4. *Let G be a graph of order p, let a, b and n be nonnegative integers such that $1 \le a < b$, and let g and f be two integer-valued functions defined on $V(G)$ such that $a \le g(x) < f(x) \le b$ for each $x \in V(G)$. If the binding number $bind(G) > \frac{(a+b-1)(p-1)}{(a+1)p-(a+b)-bn+2}$ and $p \ge \frac{(a+b-1)(a+b-2)}{a+1} + \frac{bn}{a}$, then G is a (g,f,n)-critical graph.*

In this paper, we prove the following result on (g,f,n)-critical graphs, which is an extension of Theorem 2.

Theorem 5. *Let G be a graph of order p, and let a, b, n be nonnegative integers with $2 \le a < b$ and $p \ge \frac{(a+b-2)(a+2b-3)}{a+1} + \frac{bn}{a}$. Let g, f be two integer-valued functions defined on $V(G)$ such that $a \le g(x) < f(x) \le b$ for each $x \in V(G)$. Suppose for any subset $X \subset V(G)$, we have*

$$N_G(X) = V(G), \qquad if \qquad |X| \ge \left\lfloor \frac{((a+1)(p-1)-bn)p}{(a+b-1)(p-1)} \right\rfloor; \quad or$$

$$|N_G(X)| \ge \frac{(a+b-1)(p-1)}{(a+1)(p-1)-bn}|X|, \qquad if \qquad |X| < \left\lfloor \frac{((a+1)(p-1)-bn)p}{(a+b-1)(p-1)} \right\rfloor.$$

Then G is a (g,f,n)-critical graph.

In Theorem 5, if $n = 0$, then we get the following corollary.

Corollary 1. *Let G be a graph of order p, and let a, b be nonnegative integers with $2 \le a < b$ and $p \ge \frac{(a+b-2)(a+2b-3)}{a+1}$. Let g, f be two integer-valued* *functions defined on $V(G)$ such that $a \le g(x) < f(x) \le b$ for each $x \in V(G)$. Suppose for any subset $X \subset V(G)$, we have*

$$N_G(X) = V(G) \qquad if \qquad |X| \ge \left\lfloor \frac{(a+1)p}{a+b-1} \right\rfloor; \quad or$$

$$|N_G(X)| \ge \frac{a+b-1}{a+1}|X| \qquad if \qquad |X| < \left\lfloor \frac{(a+1)p}{a+b-1} \right\rfloor.$$

Then G has a (g, f)-factor.

In Theorem 5, if $g(x) \equiv a$ and $f(x) \equiv b$, then we obtain the following corollary.

Corollary 2. *Let G be a graph of order p, and let a, b, n be nonnegative integers with $2 \le a < b$ and $p \ge \frac{(a+b-2)(a+2b-3)}{a+1} + \frac{bn}{a}$. Suppose for any subset $X \subset V(G)$, we have*

$$N_G(X) = V(G) \qquad if \qquad |X| \ge \left\lfloor \frac{((a+1)(p-1)-bn)p}{(a+b-1)(p-1)} \right\rfloor; \quad or$$

$$|N_G(X)| \ge \frac{(a+b-1)(p-1)}{(a+1)(p-1)-bn}|X| \qquad if \qquad |X| < \left\lfloor \frac{((a+1)(p-1)-bn)p}{(a+b-1)(p-1)} \right\rfloor.$$

Then G is an (a, b, n)-critical graph.

2 Preliminary lemmas

Let g, f be two nonnegative integer-valued functions defined on $V(G)$ with $g(x) < f(x)$ for each $x \in V(G)$. If $S, T \subseteq V(G)$, then we define $f(S) = \sum_{x\in S} f(x)$, $g(T) = \sum_{x\in T} g(x)$ and $d_{G-S}(T) = \sum_{x\in T} d_{G-S}(x)$. If S and T are disjoint subsets of $V(G)$ define

$$\delta_G(S,T) = f(S) + d_{G-S}(T) - g(T),$$

and if $|S| \ge n$ define

$$f_n(S) = \max\{f(U) : U \subseteq S \text{ and } |U| = n\}. \tag{1}$$

Li and Matsuda [10] obtained a necessary and sufficient condition for a graph to be a (g, f, n)-critical graph, which is very useful in the proof of Theorem 5.

Lemma 2.1. [10] *Let G be a graph, $n \geq 0$ an integer, and let g and f be two integer-valued functions defined on $V(G)$ such that $g(x) < f(x)$ for each $x \in V(G)$. Then G is a (g, f, n)-critical graph if and only if for any $S \subseteq V(G)$ with $|S| \geq n$*

$$\delta_G(S, T) \geq f_n(S),$$

where $T = \{x : x \in V(G) \setminus S, d_{G-S}(x) \leq g(x)\}$.

Lemma 2.2. *Let G be a graph of order p which satisfies the assumption of Theorem 5. Then $\delta(G) \geq \frac{(b-2)p+(a+1)+bn}{a+b-1}$.*

Proof. Let u be a vertex of G with degree $\delta(G)$. Let $Y = V(G) \setminus N_G(u)$. Clearly, $u \notin N_G(Y)$, then we have

$$(a+b-1)(p-1)|Y| \leq ((a+1)(p-1)-bn)|N_G(Y)| \leq ((a+1)(p-1)-bn)(p-1),$$

that is,

$$(a + b - 1)|Y| \leq (a + 1)(p - 1) - bn.$$

Since $|Y| = p - \delta(G)$, we get

$$(a + b - 1)(p - \delta(G)) \leq (a + 1)(p - 1) - bn.$$

Thus, we obtain

$$\delta(G) \geq p - \frac{(a + 1)(p - 1) - bn}{a + b - 1} = \frac{(b - 2)p + (a + 1) + bn}{a + b - 1}.$$

3 The Proof of Theorem 5

Now we prove Theorem 5. Suppose that a graph G satisfies the conditions of Theorem 5, but is not a (g, f, n)-critical graph. Then by Lemma 2.1, there exists a subset S of $V(G)$ with $|S| \geq n$ such that

$$\delta_G(S, T) \leq f_n(S) - 1, \tag{2}$$

where $T = \{x : x \in V(G) \setminus S,\ d_{G-S}(x) \leq g(x)\}$. We choose such subsets S and T so that $|T|$ is as small as possible.

We firstly show that the following claim holds.

Claim 1. $d_{G-S}(x) \leq g(x) - 1 \leq b - 2$ for each $x \in V(G)$.

Proof. If $d_{G-S}(x) \geq g(x)$ for some $x \in T$, then the subsets S and $T \setminus \{x\}$ satisfy (2). This contradicts the choice of S and T. Therefore, we have

$$d_{G-S}(x) \leq g(x) - 1 \leq b - 2$$

for each $x \in T$.
This completes the proof of Claim 1.

If $T = \emptyset$, then by (1) and (2), $f(S) - 1 \geq f_n(S) - 1 \geq \delta_G(S,T) = f(S)$, a contradiction. Hence, $T \neq \emptyset$. Define

$$h = \min\{d_{G-S}(x) | x \in T\}.$$

According to Claim 1, we have

$$0 \leq h \leq b-2.$$

In view of Lemma 2.2 and the definition of h, we obtain

$$|S| \geq \delta(G) - h \geq \frac{(b-2)p + (a+1) + bn}{a+b-1} - h. \tag{3}$$

Since $a \leq g(x) < f(x) \leq b$ for each $x \in V(G)$, it follows from (1) and (2) that

$$\delta_G(S,T) \leq f_n(S) - 1 \leq bn - 1 \tag{4}$$

and

$$\delta_G(S,T) = f(S) + d_{G-S}(T) - g(T) \geq (a+1)|S| + d_{G-S}(T) - (b-1)|T|,$$

so that

$$bn - 1 \geq (a+1)|S| + d_{G-S}(T) - (b-1)|T|. \tag{5}$$

In the following we shall consider three cases according to the value of h and derive a contradiction in each case.

Case 1. $h = 0$.

We define $I = \{x | x \in T, d_{G-S}(x) = 0\}$. Then I is an independent vertex subset of G and $I \neq \emptyset$. Let $Y = V(G) \setminus S$. Then $N_G(Y) \neq V(G)$ since $h = 0$. By the condition of Theorem 5, we obtain

$$p - |I| \geq |N_G(Y)| \geq \frac{(a+b-1)(p-1)}{(a+1)(p-1) - bn}|Y| = \frac{(a+b-1)(p-1)}{(a+1)(p-1) - bn}(p - |S|),$$

which implies

$$|S| \geq p - \frac{((a+1)(p-1) - bn)(p - |I|)}{(a+b-1)(p-1)}. \tag{6}$$

In view of (5), (6) and $|S| + |T| \leq p$, we have

$$\begin{aligned}
bn - 1 &\geq (a+1)|S| + d_{G-S}(T) - (b-1)|T| \\
&\geq (a+1)|S| + |T| - |I| - (b-1)|T| \\
&= (a+1)|S| - |I| - (b-2)|T| \\
&\geq (a+1)|S| - |I| - (b-2)(p - |S|) \\
&= (a+b-1)|S| - |I| - (b-2)p \\
&\geq (a+b-1)(p - \frac{((a+1)(p-1) - bn)(p - |I|)}{(a+b-1)(p-1)}) - |I| - (b-2)p \\
&= (a+1)p - \frac{((a+1)(p-1) - bn)(p - |I|)}{p-1} - |I| \\
&\geq (a+1)p - \frac{((a+1)(p-1) - bn)(p-1)}{p-1} - 1 \\
&= (a+1)p - (a+1)(p-1) + bn - 1 \\
&= bn + a,
\end{aligned}$$

which is a contradiction.

Case 2. $h = 1$.

Subcase 2.1. $|T| > \left\lfloor \frac{((a+1)(p-1)-bn)p}{(a+b-1)(p-1)} \right\rfloor$.

Clearly,

$$|T| \geq \left\lfloor \frac{((a+1)(p-1) - bn)p}{(a+b-1)(p-1)} \right\rfloor + 1. \tag{7}$$

There exists $u \in T$ such that $d_{G-S}(u) = h = 1$. Thus, we have

$$u \notin N_G(T \setminus N_G(u)). \tag{8}$$

According to (7) and $d_{G-S}(u) = 1$, we obtain

$$|T \setminus N_G(u)| \geq |T| - 1 \geq \left\lfloor \frac{((a+1)(p-1) - bn)p}{(a+b-1)(p-1)} \right\rfloor,$$

which implies that

$$N_G(T \setminus N_G(u)) = V(G).$$

This contradicts (8).

Subcase 2.2. $|T| \leq \left\lfloor \frac{((a+1)(p-1)-bn)p}{(a+b-1)(p-1)} \right\rfloor$.

Let $r = |\{x : x \in T,\ d_{G-S}(x) = 1\}|$. Obviously, $r \geq 1$ and $|T| \geq r$. In view of (3) and $h = 1$, we obtain

$$|S| \geq \frac{(b-2)p + (a+1) + bn}{a+b-1} - 1 = \frac{(b-2)(p-1) + bn}{a+b-1}. \tag{9}$$

Subcase 2.2.1. $|T| \le \frac{(a+1)(p-1)-bn}{a+b-1}$.
In this case, from (5) and (9) we have

$$\begin{aligned} bn-1 &\ge (a+1)|S| + d_{G-S}(T) - (b-1)|T| \\ &\ge (a+1)|S| + 2(|T|-r) + r - (b-1)|T| \\ &= (a+1)|S| - (b-3)|T| - r \\ &\ge \frac{(a+1)((b-2)(p-1)+bn)}{a+b-1} - \frac{(b-3)((a+1)(p-1)-bn)}{a+b-1} - r \\ &= \frac{(a+1)(p-1)-bn+(a+b-1)bn}{a+b-1} - r \\ &= bn + \frac{(a+1)(p-1)-bn}{a+b-1} - r \\ &\ge bn + |T| - r \ge bn, \end{aligned}$$

which is a contradiction.
Subcase 2.2.2. $|T| > \frac{(a+1)(p-1)-bn}{a+b-1}$.
According to (9), we obtain

$$|S| + |T| > \frac{(b-2)(p-1)+bn}{a+b-1} + \frac{(a+1)(p-1)-bn}{a+b-1} = p-1.$$

From this and $|S| + |T| \le p$, we have

$$|S| + |T| = p. \tag{10}$$

By (10) and $|T| \le \left\lfloor \frac{((a+1)(p-1)-bn)p}{(a+b-1)(p-1)} \right\rfloor \le \frac{((a+1)(p-1)-bn)p}{(a+b-1)(p-1)}$, we have

$$\begin{aligned} \delta_G(S,T) &= f(S) + d_{G-S}(T) - g(T) \\ &\ge (a+1)|S| + |T| - (b-1)|T| \\ &= (a+1)|S| - (b-2)|T| \\ &= (a+1)(p-|T|) - (b-2)|T| \\ &= (a+1)p - (a+b-1)|T| \\ &\ge (a+1)p - \frac{((a+1)(p-1)-bn)p}{p-1} \\ &= \frac{pbn}{p-1} \\ &\ge bn. \end{aligned}$$

That contradicts (4).
Case 3. $2 \le h \le b-2$.

By (5) and $|S| + |T| \leq p$, we obtain

$$\begin{aligned} bn &> bn - 1 \geq (a+1)|S| + d_{G-S}(T) - (b-1)|T| \\ &\geq (a+1)|S| + h|T| - (b-1)|T| \\ &= (a+1)|S| - (b-1-h)|T| \\ &\geq (a+1)|S| - (b-1-h)(p - |S|) \\ &= (a+b-h)|S| - (b-1-h)p, \end{aligned}$$

that is,

$$|S| < \frac{(b-1-h)p + bn}{a+b-h}. \tag{11}$$

According to (11) and $\delta(G) \leq |S| + h$, we have

$$\delta(G) \leq |S| + h < \frac{(b-1-h)p + bn}{a+b-h} + h. \tag{12}$$

Let $F(h) = \frac{(b-1-h)p+bn}{a+b-h} + h$. Then we obtain

$$\begin{aligned} F'(h) &= \frac{-p(a+b-h) + (b-1-h)p + bn}{(a+b-h)^2} + 1 \\ &= 1 - \frac{(a+1)p - bn}{(a+b-h)^2} \leq 1 - \frac{(a+1)p - bn}{(a+b-2)^2} \\ &\leq 1 - \frac{(a+b-2)(a+2b-3) + \frac{a+1}{a}bn - bn}{(a+b-2)^2} \\ &\leq 1 - \frac{a+2b-3}{a+b-2} = -\frac{b-1}{a+b-2} \\ &< 0. \end{aligned}$$

Clearly, the function $F(h)$ attains its maximum value at $h = 2$ since $2 \leq h \leq b - 2$. Then we have

$$F(h) \leq F(2) = \frac{(b-3)p + bn}{a+b-2} + 2. \tag{13}$$

According to Lemma 2.2, (12) and (13), we obtain

$$\frac{(b-2)p + (a+1) + bn}{a+b-1} \leq \delta(G) < \frac{(b-3)p + bn}{a+b-2} + 2,$$

which implies that

$$p < \frac{(a+b-2)(a+2b-3) + bn}{a+1} \leq \frac{(a+b-2)(a+2b-3)}{a+1} + \frac{bn}{a},$$

this contradicts $p \geq \frac{(a+b-2)(a+2b-3)}{a+1} + \frac{bn}{a}$.

From the argument above, we deduce the contradictions. Hence, G is a (g, f, n)-critical graph.

Completing the proof of Theorem 5.

4 Remark

Let us show that the condition in Theorem 5 can not be replaced by the condition that $N_G(X) = V(G)$ or $|N_G(X)| \geq \frac{(a+b-1)(p-1)}{(a+1)(p-1)-bn}|X|$ for all $X \subseteq V(G)$. Let $a \geq 2$, $b = a+1$ and $n \geq 0$ be integers and b is odd. Let m be any odd positive integer. We construct a graph G of order p as follows. Let $V(G) = S \cup T$ (disjoint union), $|S| = (a-1)m + n$ and $|T| = bm + 1$, and put $T = \{t_1, t_2, \cdots, t_{2l}\}$, where $2l = bm + 1$. For each $s \in S$, define $N_G(s) = V(G) \setminus \{s\}$, and for any $t \in T$, define $N_G(t) = S \cup \{t'\}$, where $\{t, t'\} = \{t_{2i-1}, t_{2i}\}$ for some i, $1 \leq i \leq l$. Clearly, $p = (a-1)m + n + bm + 1$. We first show that the condition that $N_G(X) = V(G)$ or $|N_G(X)| \geq \frac{(a+b-1)(p-1)}{(a+1)(p-1)-bn}|X|$ for all $X \subseteq V(G)$ holds. Let any $X \subseteq V(G)$. It is obvious that if $|X \cap S| \geq 2$, or $|X \cap S| = 1$ and $|X \cap T| \geq 1$, then $N_G(X) = V(G)$. Of course, if $|X| = 1$ and $X \subseteq S$, then $|N_G(X)| = |V(G)| - 1 = p - 1 > \frac{(a+b-1)(p-1)}{bm(a+b-1)} = \frac{(a+b-1)(p-1)}{b(a-1)m+bn+b^2m-bn} = \frac{(a+b-1)(p-1)}{b((a-1)m+n+bm)-bn} = \frac{(a+b-1)(p-1)}{b(p-1)-bn} = \frac{(a+b-1)(p-1)}{(a+1)(p-1)-bn} = \frac{(a+b-1)(p-1)}{(a+1)(p-1)-bn}|X|$. Hence we may assume $X \subseteq T$. Since $|N_G(X)| = |S| + |X| = (a-1)m + n + |X|$, $|N_G(X)| \geq \frac{(a+b-1)(p-1)}{(a+1)(p-1)-bn}|X|$ holds if and only if $(a-1)m + n + |X| \geq \frac{(a+b-1)(p-1)}{(a+1)(p-1)-bn}|X|$. This inequality is equivalent to $|X| \leq bm$. Thus if $X \neq T$ and $X \subset T$, then $|N_G(X)| \geq \frac{(a+b-1)(p-1)}{(a+1)(p-1)-bn}|X|$ for all $X \subseteq V(G)$ holds. If $X = T$, then $N_G(X) = V(G)$. Consequently, $N_G(X) = V(G)$ or $|N_G(X)| \geq \frac{(a+b-1)(p-1)}{(a+1)(p-1)-bn}|X|$ for all $X \subseteq V(G)$ follows. In the following, we show that G is not a (g, f, n)-critical graph. For above S and T, obviously, $|S| > n$ and $d_{G-S}(t) = 1$ for each $t \in T$. Since $a \leq g(x) < f(x) \leq b$ and $b = a+1$, then we have $g(x) = a$ and $f(x) = b = a+1$ for each $x \in V(G)$. Thus, we obtain

$$\begin{aligned} \delta_G(S,T) &= f(S) + d_{G-S}(T) - g(T) \\ &= b|S| + |T| - a|T| \\ &= b|S| - (a-1)|T| \\ &= b((a-1)m + n) - (a-1)(bm+1) \\ &= bn - a + 1 \leq bn - 1 < bn = f_n(S). \end{aligned}$$

By Lemma 2.1, G is not a (g, f, n)-critical graph. In the above sense, the condition in Theorem 5 is best possible.

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Jiangsu University of Science and Technology
School of Mathematics and Physics
Mengxi Road 2, Zhenjiang, Jiangsu 212003, People's Republic of China
Email: zsz_cumt@163.com