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On residuated n-lattice

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Abstract

In this paper, we introduce residuated n-lattice: a variety of residuated semigroup equipped with binary hyperoperations n-sup and n-inf. We define the left bound, right bound, n-supremum, n-infimum, maximum and minimum with respect to it′s relation. By these way, the notion of residuated semigroup has been generalized. Some examples of residuated n-lattice have been gave, then we show that our definition is an extention of the old ones. Also, we state and prove some theorems in this structure.

1 Introduction

Various logical algebras have been proposed as the semantical systems of non-classical logic systems, for example, residuated lattices, MV-algebras, BL-algebras, Gödel algebras, lattice implication algebras, MTL-algebras, NM-algebras and R0-algebras, etc. Among these logical algebras, residuated lattices are very basic and important algebraic structures because the other logical algebras are all particular cases of residuated lattices. The concept of a commutative residuated lattice was firstly introduced by M. Ward and R.P. Dilworth as generalization of ideal lattices of rings [8]. The properties of a commutative residuated lattice were presented in [3]. Non-commutative residuated lattices, called sometimes pseudo-residuated lattices, biresiduated lattices or generalized residuated lattices are algebraic counterpart of substructural logics, that is, logics which lack some of the three structural rules,

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namely contraction, weakening and exchange. Complete studies on residuated lattices were developed by H. Ono, T. Kowalski, P. Jipsen and C. Tsinakis.

The theory of semigroup has been intensively studied for more than four and half decades, and a huge and very diverse material has been accumulated here. A systematic overview of this material was due already long ago. The first of them [7] is devoted to equational aspects of the theory of semigroup varieties. The subject of the second article [6] was related to consideration of structural properties of semigroups in varieties such as local finiteness and residual finiteness, decompositions into bands and embeddings. More than 200 papers devoted (completely or partially) to lattices of semigroup varieties have been published so far. T.S. Blyth studied the notions of residuated semigroups, an ordered semigroup S in which all translations λ_x and ρ_x are residuated [1].

The hyperstructure theory was introduced by Marty , at the 8th Congress of Scandinavian Mathematicians. In his definition, a function $f : A \times A \to P^*(A)$, of the set $A \times A$ into the set of all nonempty subsets of A, is called a binary hyperoperation, and the pair (A, f) is called a hypergroupoid. If f is associative, A is called a semihypergroup, and it is said to be commutative if f is commutative. Also, an element $1 \in A$ is called the unit or the neutral element if $a \in f(1, a)$, for all $a \in A$.

Residuated lattices play key role in the study of fuzzy logics and the associated algebraic structures. In Mathematics, the supremum, infimum, maximum and minimum are defined for subsets of partially ordered sets. These notions are important in analysis, algebra, geometry, applied Mathematics and other science. With this background in mind, the current research aimed to schedule the panorama of formal fuzzy logics more specifically in order, to develop the notion of residuated semigroup and define residuated n-lattice. Residuated n-lattice behave differently, and we attempted to state and prove the propositions and theorems that determine the properties of these structures. With this new structure, we can get a better understanding of other algebraic structures. By presenting the properties of this structure, we can obtain the application of residuated n-lattice in other sciences. Just as residuated lattices are substructure of many important algebraic structures, such as MTL-algebra, BL-algebra, MV-algebra and etc., residuated n-lattice can also be a prelude to the construction and introduction of new structures.

In the present study, we basically assessed residuated semigroups, which is an important tool in fuzzy mathematics. Evaluation of residuated semigroups, resulted in the better cognition of this structure. Therefore, we define the left bound, right bound, maximum and minimum and two hyperoperators n-supremum, n-infimum for subset S of the residuated semigroup X with respect to the order of X. By means of these notions, we define hyper residuated n-lattice and study them in detail. A basic tool in algebras is the notion of a mapping. In the theory of ordered sets there is the corresponding concept of a residuated mapping. By means of residuated mapping, we give some new results.

2 Preliminaries

Definition 2.1. *[1] A residuated lattice is an algebra $\mathcal{L} = (L, \vee, \wedge, *, \Rightarrow, 0, 1)$ with four binary operations and two constant 0,1 such that:*

– $(L, \vee, \wedge, 0, 1)$ is a bounded lattice,

– $*$ is commutative and associative, with 1 as neutral element, and

– $x * y \leq z$ if and only if $x \leq (y \Rightarrow z)$, for all x, y and z in L (residuation principle).

Definition 2.2. *[10] By a hyper residuated lattice we mean a non-empty partially ordered set $(L, \leq)$ endowed with four binary hyperoperations $\vee, \wedge, *, \Rightarrow$ and two constants 0 and 1 satisfying the following conditions:*

– $(L, \leq, \vee, \wedge, 0, 1)$ is a bounded superlattice,

– $(L, *, 1)$ is a commutative semihypergroup with 1 as the identity,

– $a * c \ll b$ if and only if $c \ll a \Rightarrow b$,

where $A \ll B$ means that there exist $a \in A$ and $b \in B$ such that $a \leq b$, for all non-empty subsets A and B of L.

Definition 2.3. *[1] A semigroup is an algebraic structure consisting of a set together with an associative binary operation.*

Definition 2.4. *[1] An ordered semigroup is a semigroup $(S, \cdot)$ together with a partial order R that is compatible with the semigroup operation, meaning that xRy implies $z \cdot xRz \cdot y$ and $x \cdot zRy \cdot z$ for all $x, y, z \in S$.*

Definition 2.5. *[1] By a residuated semigroup we shall mean an ordered semigroup S in which all translations $\lambda_x : y \to xy$ and $\rho_x : y \to yx$ are residuated.*

If S is a residuated semigroup then the quasi-residuals $x \to y$ and $x \rightsquigarrow y$ are principal down-sets. In what follows we shall denote their respective top elements by $x \to y$ and $x \rightsquigarrow y$. Consequently,

$$x \to y = \lambda_y^+(x) = max\{z \in S \mid yz \; R \; x\},$$
$$x \rightsquigarrow y = \rho_y^+(x) = max\{z \in S \mid zy \; R \; x\}.$$

Definition 2.6. *[1] If (A, R_1) and (B, R_2) are ordered sets, then we say that a mapping $f : A \to B$ is isotone if*

$$\textit{If } (x \; R_1 \; y), \textit{ then } (f(x) \; R_1 \; f(y)), \textit{ for all } x, y \in A.$$

Definition 2.7. *[1] A mapping $f : E \to F$ is said to be residuated, if f is isoton and there is an isotone mapping $g : F \to E$ such that $f \circ g \; R \; id_F$ and $g \circ f \; R \; id_E$.*

We note in particular that if $f : E \to F$ is a residuated mapping then an isotone mapping $g : F \to E$ which is such that $g \circ f \; R \; id_E$ and $f \circ g \; R \; id_F$ is in fact unique. To see this, suppose that g and g^* are each isotone and satisfy these properties. Then $g = id_E \circ g \; R \; (g^* \circ f) \circ g = g^* \circ (f \circ g) \; R \; g^* \circ id_F = g^*$. Similarly, $g^* \; R \; g$ and therefore $g = g^*$.

We shall denote this unique g by f^+ and call it the residual of f.

It is clear from the above that $f : E \to F$ is residuated if and only if, for every $y \in F$, there exists

$$f^+(y) = max\{x \in E \mid f(x) \; R \; y\}.$$

Moreover, $f^+ \circ f \; R \; id_E$ and $f \circ f^+ \; R \; id_F$.

Theorem 2.1. *[1] If $f : E \to F$ and $g : F \to G$ are residuated mappings then so is $g \circ f : E \to G$ and $(g \circ f)^+ = f^+ \circ g^+$.*

3 Construction of residuated n-lattices

Definition 3.1. *Let X be a residuated semigroup that ordered with relation R and $S \subseteq X$. Then*

- *$r \in X$ is called a right bound for S (with respect to relation R) if $s \; R \; r$, for all $s \in S$. We denote by $\mathcal{R}(S)$ the set of all right bounds of S.*
- *$l \in X$ is called a left bound for S (with respect to relation R) if $l \; R \; s$, for all $s \in S$. We denote by $\mathcal{L}(S)$ the set of all right bounds of S.*
- *$b \in X$ is called a bound for S (with respect to relation R) if $e \in \mathcal{R}(S) \cap \mathcal{L}(S)$, in the other words, $s \; R \; b$ and $b \; R \; s$, for all $s \in S$. We denote by $\mathcal{B}(S)$ the set of all bounds of S.*
- *We define hyperoperators n-supremum and n-infimum of S (with respect to relation R) as follows:*

$$n\text{-}sup(S) := \{s \in \mathcal{R}(S) \mid s \in \mathcal{L}(\mathcal{R}(S)) = \mathcal{R}(S) \cap \mathcal{L}(\mathcal{R}(S))\},$$
$$n\text{-}inf(S) := \{s \in \mathcal{L}(S) \mid l \in \mathcal{R}(\mathcal{L}(S)) = \mathcal{L}(S) \cap \mathcal{R}(\mathcal{L}(S))\}.$$

- *We define maximum and minimum of S as follows:*

$$max(S) := S \cap \; (n\text{- } sup(S)),$$
$$min(S) := S \cap \; (n\text{-}inf(S)).$$

$max(S)$ and $min(S)$ are hyper operators.

Remark 3.1. *It is clear that n-inf$(S) \; R \; S$ and n-inf$(S) \; R \; S$.*

Example 3.1. *Clearly* $(\mathbb{N}, +, \leq)$ *is an ordered semigroup. If* $S = \{1, 2, 3, 4\} \subseteq \mathbb{N}$*, then we get* n*- sup*(S) *and* n*- inf*(S) *as follows:*

$n - sup$	1	2	3	4
1	$\{1,2,3,4\}$	$\{2,3,4\}$	$\{3,4\}$	$\{4\}$
2	$\{2,3,4\}$	$\{2,3,4\}$	$\{3,4\}$	$\{4\}$
3	$\{3,4\}$	$\{3,4\}$	$\{3,4\}$	$\{4\}$
4	$\{4\}$	$\{4\}$	$\{4\}$	$\{4\}$

$n{-}inf$	1	2	3	4
1	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
2	$\{1\}$	$\{1,2\}$	$\{1,2\}$	$\{1,2\}$
3	$\{1\}$	$\{1,2\}$	$\{1,2,3\}$	$\{1,2,3\}$
4	$\{1\}$	$\{1,2\}$	$\{1,2,3\}$	$\{1,2,3,4\}$

Definition 3.2. *Let* $n \in \mathbb{N}$ *and* X *be a residuated semigroup.* X *is said to be residuated* n*-lattice if for every subset* S *of* X *with* $Card(S) = n$*, we have* n*-sup*$(S) \neq \emptyset$ *and* n*-inf*$(S) \neq \emptyset$.

Remark 3.2. *In Definition 3.2, if* $n = 2$*, then we get the definition of residuated lattice.*

Remark 3.3. *Residuated* n*-lattices are special types of hyper residuated lattices. In Definition 3.2, if we define* $xy = \{x * y\}$ *and* $x \to y = \{x \Rightarrow y\}$*, then we get the definition of hyper residuated lattice. It is clear that every hyper residuated lattice is not residuated* n*-lattice.*

Example 3.2. *If* $(G, *, \leq)$ *is an ordered group with identity element* e*, we define* $x \to y = y^{-1}x$ *and* $x \rightsquigarrow y = xy^{-1}$*, for all* $x, y \in G$*. For every subgroup* S *of* G *which* $Card(S) = n$*,* $(G, *, \leq, \to, \rightsquigarrow,$ n*-sup*(S)*,* n*-inf*(S)*,*$\{e\}, G)$ *is a residuated* n*-lattice.*

Example 3.3. *If* S *is a semigroup, then* $(\mathbb{P}(S), \cdot, \subseteq)$ *is an ordered semigroup under the multiplication* $X \cdot Y = \{xy \mid x \in X, y \in Y\}$*. Clearly, the ordered semigroup* $(\mathbb{P}(S), \cdot, \subseteq)$ *is residuated. We have* $X \to Y = \{z \in S \mid (\forall y \in Y) yz \in X\}$ *and* $X \rightsquigarrow Y = \{z \in S \mid (\forall y \in Y) zy \in X\}$*. Clearly* $(\mathbb{P}(S), \cdot, \subseteq, \to, \rightsquigarrow,$ n*-sup*(S)*,* n*-inf*(S)*,*$\emptyset, S)$ *is a residuated* n*-lattice.*

Example 3.4. *Let* $(L = \{0, a, b, 1\}, \leq)$ *be a chain such that* $0 < a < b < 1$*. We have two hyperoperations* n*-sup*(S) *and* n*-inf*(S) *on* L *by the following tables:*

$n - sup$	0	a	b	1
0	$\{0,a,b,1\}$	$\{a,b,1\}$	$\{b,1\}$	$\{1\}$
a	$\{a,b,1\}$	$\{a,b,1\}$	$\{b,1\}$	$\{1\}$
b	$\{b,1\}$	$\{b,1\}$	$\{b,1\}$	$\{1\}$
1	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$

$n{-}inf$	0	a	b	1
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{0\}$	$\{0,a\}$	$\{0,a\}$	$\{0,a\}$
b	$\{0\}$	$\{0,a\}$	$\{0,b,a\}$	$\{0,b,a\}$
1	$\{0\}$	$\{0,a\}$	$\{0,b,a\}$	$\{0,b,a,1\}$

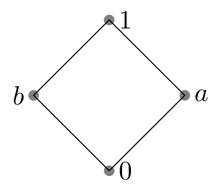
Let $x\cdot y = n$-$inf(x,y)$, for any $x,y\in L$ be the binary hyperoperation on L. Consider the following tables:

$\to$	0	a	b	1
0	1	1	1	1
a	0	1	1	1
b	0	a	1	1
1	0	a	b	1

$\rightsquigarrow$	0	a	b	1
0	1	1	1	1
a	0	1	1	1
b	0	a	1	1
1	0	a	b	1

It is clear that $(L$, n-inf, n-$sup,\cdot,\to,\rightsquigarrow 0,1)$ is a residuated 4-lattice.

Example 3.5. *Let $L=\{0,a,b,1\}$ be a partially ordered set having the following Hasse diagram*



Define n-$sup(x,y)=\{max\{x,y\}\}$, $x\cdot y = n$-$inf(x,y)=\{min\{x,y\}\}$. So we have the following tables:

$n-sup$	0	a	b	1
0	$\{0,a,b,1\}$	$\{a,1\}$	$\{b,1\}$	$\{1\}$
a	$\{a,1\}$	$\{a,1\}$	$\{1\}$	$\{1\}$
b	$\{b,1\}$	$\{1\}$	$\{b,1\}$	$\{1\}$
1	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$

$n{-}inf$	0	a	b	1
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{0\}$	$\{0,a\}$	$\{0\}$	$\{0,a\}$
b	$\{0\}$	$\{0\}$	$\{0,b\}$	$\{0,b\}$
1	$\{0\}$	$\{0,a\}$	$\{0,b\}$	$\{0,b,a,1\}$

Now, consider the following diagram.

$\to$	0	a	b	1
0	1	1	1	1
a	b	1	b	1
b	a	a	1	1
1	0	a	b	1

$\rightsquigarrow$	0	a	b	1
0	1	1	1	1
a	b	1	b	1
b	a	a	1	1
1	0	a	b	1

Clearly, $(A, \cdot, \leq, \to, \rightsquigarrow,$ *n-sup*(S), *n-inf*(S),$0, 1)$ *is a residuated 4-lattice. Let* $L = \{0, b, 1\}$, *define n-sup*$(x, y) = \{max\{x, y\}\}$, $x{\cdot}y =$ *n-inf*$(x, y) = \{min\{x, y\}\}$. *Then*

$\Rightarrow$	0	b	1
0	1	1	1
b	b	1	1
1	0	b	1

$\hookrightarrow$	0	b	1
0	1	1	1
b	b	1	1
1	0	b	1

$(\{0, b, 1\}, \cdot, \leq, \Rightarrow, \hookrightarrow,$ *n-sup*(S), *n-inf*(S),$0, 1)$ *is a residuated 3-lattice,*

Example 3.6. *In Example 3.5,* $S = \{0, a, b\}$ *is not a residuated n-lattice since n-sup*$(S) = \emptyset$.

Remark 3.4. *In Example 3.5, we see that residuated n-lattice do not have any relationship to residuated* $(n + 1)$*-lattice*

Theorem 3.1. *If* $(X, \cdot, R, \to, \rightsquigarrow,$ *n-sup*(S), *n-inf*(S),$0, 1)$ *is a residuated n-lattice with* 0 *as smallest and* 1 *as greatest element, then the following statements are equivalent:*

(i) X *is commutative,*

(ii) $x \to y = x \rightsquigarrow y$, *for all* $x, y \in X$.

Proof. Observe that (i) is equivalent to saying that $\lambda_y = \rho_y$ for every $y \in S$. By the uniqueness of residuals, this is equivalent to $\lambda_y^+ = \rho_y^+$ which is equivalent to (ii). □

Theorem 3.2. *If* $(X, \cdot, R, \to, \rightsquigarrow,$ *n-sup*(S), *n-inf*(S),$0, 1)$ *is a residuated n-lattice, then for all* $x, y, z \in X$,

(1) $(x(y \to x))\ \ R\ \ y$.

(2) $((y \rightsquigarrow x)x)\ \ R\ \ y$.

(3) $y\ \ R\ \ (xy \to x)$.

(4) $y\ \ R\ \ (yx \rightsquigarrow x)$.

(5) $x\ \ R\ \ (y \rightsquigarrow (y \to x))$.

(6) $x\ \ R\ \ (y \to (y \rightsquigarrow x))$.

(7) $(x \to y) \to z = x \to yz$.

(8) $(x \rightsquigarrow y) \rightsquigarrow z = x \rightsquigarrow zy$.

(9) $(x \to y) \rightsquigarrow z = (x \rightsquigarrow z) \to y$.
(10) $((x \to y)z)\ R\ (xz \to y)$.
(11) $(y(x \rightsquigarrow z))\ R\ (yx \rightsquigarrow z)$.

Proof. (1), (2), (3) and (4) follow from the definitions of $x \to y$ and $x \rightsquigarrow y$. By (1), (2), we get (5) and (6).

(7) Since X is a semigroup we have $\lambda_y \circ \lambda_z = \lambda_{yz}$ which, by Theorem 2.1, gives $\lambda_z^+ \circ \lambda_y^+ = \lambda_{yz}^+$.

(8) Similar (7), $\rho_y \circ \rho_z = \rho_{zy}$ gives (8).

(9) Likewise (7), $\lambda_y \circ \rho_z = \rho_z \circ \lambda_y$ gives (9).

(10) If E is an ordered set and $f, g \in ResE$, then

$$[(f \circ g)\ R\ (g \circ f)] \text{ if and only if } [(g \circ f^+)\ R\ (f^+ \circ g)].$$

In fact, if $(f \circ g)\ R\ (g \circ f)$, then $(g \circ f^+)\ R\ (f^+ \circ f \circ g \circ f^+)\ R\ (f^+ \circ g \circ f \circ f^+)\ R\ (f^+ \circ g)$.

Conversely, if $(g \circ f^+)\ R\ (f^+ \circ g)$, then $(f \circ g)\ R\ (f \circ g \circ f^+ \circ f)\ R\ (f \circ f^+ \circ g \circ f)\ R\ (g \circ f)$.

Writing $\lambda_y \circ \rho_z = \rho_z \circ \lambda_y$ as $(\lambda_y \circ \rho_z)\ R\ (\rho_z \circ \lambda y)$ and $(\rho_z \circ \lambda_y)\ R\ (\lambda_y \circ \rho_z)$, we obtain from this observation the inequalities $(\rho_z \circ \lambda_y^+)\ R\ (\lambda_y^+ \circ \rho_z)$ and $(\lambda_y \circ \rho_z^+)\ R\ (\rho_z^+ \circ \lambda_y)$ which are (10).

(11) It is clear by (10).

□

Theorem 3.3. *If $(X, \cdot, R, \to, \rightsquigarrow$, n-sup(S), n-inf$(S), 0, 1)$ is a residuated n-lattice and $x \to y = x \rightsquigarrow y$, then for all $x, y \in X$,*
(1) $1 \to x = x$, $x \to x = 1$.
(2) $(x \cdot y\ R\ x, y)$ *and* $(x \cdot 0 = 0)$, *and* $y\ R\ x \to y$.
(3) $x \cdot y\ R\ x \to y$.
(4) $x\ R\ y$ *iff* $x \to y = 1$.
(5) $x \to y = y \to x = 1$ *if and only if* $x = y$ *and* $x \to 1 = 1$, *and* $0 \to x = 1$.

Proof. (1) Since $x \cdot 1 = x\ R\ x$ so $x\ R\ (1 \to x)$. If we have $z \in X$ such that $1 \cdot z = x$, then $z\ R\ x$ and so $\{x\} = n\text{-sup}\{z \in X \mid 1 \cdot z\ R\ x\} = \{1 \to x\}$.

From $1 \cdot x = x\ R\ x$, so $1\ R\ x \to x$. Since $x \to x\ R\ 1$ thus $x \to x = 1$.

(2) Follows from (1) and Theorem 3.1, $x \cdot y\ R\ y$ thus $y\ R\ x \to y$.

(3) Follows from (1) and (2), $x \cdot y\ R\ y$ and $y\ R\ x \to y$ so $x \cdot y\ R\ x \to y$.

(4) We have $x\ R\ y$ if and only if $x \cdot 1\ R\ y$ if and only if $1\ R\ x \to y$ if and only if $x \to y = 1$.

(5) Follows from (4).

□

Theorem 3.4. *If X is a residuated n-lattice and $x, y \in X$, then*
(1) $x(y \to x) = y$ *if and only if* $(\exists z \in X)\ \ y = xz$.
(2) $(y \rightsquigarrow x)x = y$ *if and only if* $(\exists z \in X)\ \ y = zx$.
(3) $y = xy \to x$ *if and only if* $(\exists z \in X)\ \ y = z \to x$.
(4) $y = yx \rightsquigarrow x$ *if and only if* $(\exists z \in X)\ \ y = z \rightsquigarrow x$.
(5) $y = x \rightsquigarrow (x \to y)$ *if and only if* $(\exists z \in X)\ \ y = x \rightsquigarrow z$.
(6) $y = x \to (x \rightsquigarrow y)$ *if and only if* $(\exists z \in X)\ \ y = x \to z$.

Proof. For parts (1)-(4), we have:
If $f : A \to B$ is a residuated mapping, then

$$x = ff^+(x) \text{ if and only if } (\exists y \in A)\ \ x = f(y),$$
$$x = f^+f(x) \text{ if and only if } (\exists y \in B)\ \ x = f^+(y).$$

Applying these observations to λ_x and ρ_x we obtain (1)-(4).
Also, for parts (5) and (6), for all $x, y, z \in X$ we have:

$$(z\ \ R\ \ (x \to y)) \text{ if and only if } ((yz)\ \ R\ \ x) \text{ if and only if } (y\ \ R\ \ (x \rightsquigarrow z)).$$

Thus the mapping $\eta_x : X \to X^d$ given by the prescription $\eta_x(y) = x \to y$ is residuated with η_x^+ given by $\eta_x^+(z) = x \rightsquigarrow z$. Using this observations to η_x we obtain (5) and (6). □

Conclusion and future work

The notion of residuated lattice was introduced by M. Ward and R. P. Dilworth in 1939. The first published description of semigroup was given by E. Lyapin in 1935. Many researchers have worked on these two issues.

This article was motivated by previous research on residuated semigroups. Using the notion of maximum, n-infimum, n-supremum and infimum, we introduced the concept of residuated n-lattices. We have defined the notion of residuated n-lattice, a new algebraic structure of a residuated lattice. Some properties of hyper residuated n-lattice had been gave and considered this structure in details.

Since the classification of algebraic structures leads to a better understanding of them, in our future work, we will continue our study of algebraic properties of residuated lattice semigroup, also we will construct quotient residuated lattices semigroup, with the view to identify a classification for these structures.

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