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Recent advances of crack propagation in human bone

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Abstract

Recent results in mathematical modeling predicting crack behavior under various load conditions in human bones as anisotropic elastic composite materials are presented in this survey. New and interesting challenges in theoretical models of fracture were proposed and had significant importance for fracture mechanics. Our goal is to present an overview of the use and limitations of existing relevant theories. The present study aims to introduce mathematical models to researchers unfamiliar with the concepts, to improve and provide new insights into bone fracture mechanics.

1 Introduction

This survey reports recent advances in our understanding of mathematical modeling providing crack behavior in human bone, being a complex material, a hard connective tissue, forming a rigid skeleton, and aims to illustrate pertinent research on the crack propagation theory in 2D bones, considered composite materials. Bone has a great hierarchized material structure, which is complex, multiphasic, heterogeneous, and anisotropic. The elastic coefficients depend on the applied load or the orientation. Bone defects, *i.e.* holes, and cracks, constitute. A common problem in medicine represents the occurrence of cracks, or other defects in bone microstructure, considered to be orthotropic elastic composite, (see [1]-[3]).

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There are excellent books and papers covering bone mechanics (see [1]-[12]), regarded as anisotropic materials. In the last period fracture mechanics studies of elastic composites in biomechanics were done by several authors, (see [13]-[17]).

An ample survey of the mathematical models of crack propagations in bones under the classical or mixed modes of fracture is done by extending fracture criteria. Crack path propagation direction and the value of the critical stress which produces crack propagation are presented (see [10]-[17]). A significant number of studies of fracture bone mechanics have been done to a very sophisticated level and reported on the effects of bone density, aging, and disease on these properties, (see [4], [18]-[19]).

Figure 1: Ilium bones

2 Developments in the theory of fracture of bone

Many mathematical models were developed for the fracture of bone phenomena, as composite material. In many papers, human bone is studied as an orthotropic composite with own microstructure (see [1]-[2], [5]-[6], [10]-[12]).

In this research work, we focus study to 2D quasistatic crack initialization in human bones, modeled as homogeneous orthotropic materials.

Craciun et al. (see [10,11]), and respectively, Baesu et al. (see [12]), formulated and studied the mathematical problem for a crack in modes I and III, respectively in mixed mode (I+II) of classical fracture in Ilium, Tibia and Femur Bones considered to be orthotropic homogeneous materials (see [2]). Were obtained stress and displacement fields and their asymptotic expressions necessary to apply the extended versions of generalized Sih's strain energy density and maximum stress criteria to get the direction of the crack initialization in the Femur, Tibia, and Ilium bones.

2.1 Representation of the fields by complex potentials. Asymptotic values

Supposing that the bone is an initial deformed homogeneous orthotropic material, with a crack of length $2a$ lying on the Ox_1, modeled as a cut with two faces. Also, we consider that the configuration of the body is homogeneous and stable, *ie.* the phenomenon of resonance could not appear, (see [13]).

2.1.1 Plane state

Craciun et al. (see [11]) and Baesu et al. (see [12]) studied a cracked bone in plane state relative to the plane Ox_1x_2, *ie.* the nonvanishing displacement components are $u_j = u_j\,(x_1, x_2)$, for $j = 1, 2$ and $u_3 = 0$. The nonvanishing stress and displacement components, (see [11-13]) are given by

$$\begin{array}{l} t_{ij} = 2(-1)^{i+j} Re\Theta_{ij}, \quad u_i = 2Re\Upsilon_i, \\ \Theta_{11} = \alpha_1\nu_1^2\phi_1'(\zeta_1) + \alpha_2\nu_2^2\phi_2'(\zeta_2),\ \Theta_{12} = \nu_1\phi_1'(\zeta_1) + \nu_2\phi_2'(\zeta_2), \\ \Theta_{21} = \alpha_1\nu_1\phi_1'(\zeta_1) + \alpha_2\nu_2\phi_2'(\zeta_2),\ \Theta_{22} = \phi_1'(\zeta_1) + \phi_2'(\zeta_2), \\ \Upsilon_1 = \beta_1\phi_1(\zeta_1) + \beta_2\phi_2(\zeta_2),\ \Upsilon_2 = \gamma_1\phi_1(\zeta_1) + \gamma_2\phi_2(\zeta_2), \end{array} \tag{1}$$

where $\phi_j = \phi_j(\zeta_j), \zeta_j = x_1 + \nu_j x_2$ and the coefficients $\alpha_j, \beta_j\ \gamma_j,\ i, j = 1, 2$ depend by instantaneous elasticities $w_{klmn},\ k, l, m, n = 1, 2, 3$ and the roots $\nu_j,\ j = 1, 2$ of the characteristic equation.

Using Eqs. (1) and the boundary conditions we get two Riemann-Hilbert problems with the solutions $\psi_j(\zeta_j) = \phi_j'(\zeta_j), j = 1, 2,$

- for mode I (see [11], [13]):

$$\psi_j(\zeta_j) = (-1)^j \iota_j I_j,\ j = 1, 2 \tag{2}$$

where

$$\iota_1 = -\frac{p\alpha_2\nu_2}{2\pi\Delta\sqrt{\zeta_1^2 - a^2}}, \iota_2 = \frac{p\alpha_1\nu_1}{2\pi\Delta\sqrt{\zeta_2^2 - a^2}}, I_j = \int_{-a}^{a} \frac{\sqrt{a^2 - t^2}}{t - \zeta_j} dt, \tag{3}$$

with the following asymptotical values in a small vicinity of the crack tips

$$\psi_1(\zeta_1) = \frac{\alpha_2\nu_2\kappa_I}{\chi_1(\varphi)},\ \psi_2(\zeta_2) = -\frac{\alpha_1\nu_1\kappa_I}{\chi_1(\varphi)},\ \kappa_s = \frac{K_j}{2\Delta\sqrt{2\pi r}},\ s = I, II \tag{4}$$

Respectively,

- for mixed mode (I+II) (see [12]):

$$\begin{gathered}\psi_1(\zeta_1) = (-\iota_1/\alpha_2\nu_2)(I_1 \sin\beta\cos\beta + \mathfrak{I}_1 \sin^2\beta), \\ \psi_2(\zeta_2) = (\iota_2/\alpha_1\nu_1)(I_2 \sin\beta\cos\beta + \mathfrak{I}_2 \sin^2\beta), \\ \mathfrak{I}_j = \int_{-a}^{a} \tfrac{1}{t-\zeta_j} dt,\ j = \overline{1,3}\end{gathered} \tag{5}$$

with the asymptotical values

$$\psi_1(\zeta_1) = \frac{\alpha_2\nu_2\kappa_I + \kappa_{II}}{\chi_1(\varphi)},\ \psi_2(\zeta_2) = -\frac{\alpha_1\nu_1\kappa_I + \kappa_{II}}{\chi_1(\varphi)},\ \kappa_i = \frac{K_i}{2\Delta\sqrt{2\pi r}},\ i = I, II \tag{6}$$

where $\chi_j(\varphi) = \sqrt{\cos\varphi + \nu_j \sin\varphi}$, $j = \overline{1,3}$, p represents the normal force acting on the crack, K_I, K_{II}, and K_{III} represent the stress intensity factors in the fracture modes $I - III$ and β is the angle between the crack and Ox_2 axis.

2.1.2 Anti-plane state. Mode III of Fracture

Craciun et al. (see [10]) studied a cracked bone in an anti-plane state relative to the plane Ox_1x_2, *ie.* the only nonvanishing displacement component is $u_3 = u_3(x_1, x_2)$. We have the following representation of the stresses and displacements fields, (see [10], [13]), by an arbitrary complex potential $\phi_3 = \phi_3(\zeta_3)$, $\zeta_3 = x_1 + \nu_3 x_2 = i(\Omega_{1331}/\Omega_{2332})^{1/2}$

$$t_{23} = -2Re\phi_3'(\zeta_3), t_{13} = 2Re\nu_3\phi_3'(\zeta_3), u_3 = -2\Omega_{2332}^{-1} Re\nu_3^{-1}\phi_3(\zeta_3), \tag{7}$$

where ν_3 is the root of the characteristic equation.

Using the representation formulae and the boundary conditions it was obtained the solution to Riemann-Hilbert problems (see [10], [13]):

$$\psi(\zeta_3) = \frac{\tau}{2\pi\sqrt{\zeta_3^2 - a^2}}\mathfrak{I}_3, \tag{8}$$

with the following asymptotical value in a small vicinity of the crack tips

$$\phi(\zeta_3) = -K_{III}\sqrt{\frac{r}{2\pi}}\chi_3(\varphi), \tag{9}$$

where τ is a constant anti-plane force.

2.2 Crack propagation criteria

2.2.1 Sih's generalized fracture strain energy density criterion (SED)

We denote by W the involved strain energy density and we have

$$W(r,\varphi) = \frac{K_I^2}{4\pi r} s_i(\varphi) + \text{a regular part} \tag{10}$$

where $s_i(\varphi),\ i = I, III$ are Sih's strain energy density factors corresponding to the plane $i = I$ and respectively anti-plane modes $i = III$, (see Table 1):

The Strain Energy Density (SED) criterion states that the crack will start to propagate when the strain energy density factor $s_I(\varphi)$ has the lowest value.

Erdogan and Sih's Maximum Tangential Stress (MTS) criterion states that: The crack propagation will initialize in a radial direction in the plane normal to the direction of highest tension.
These hypotheses imply that the crack will start in a perpendicular direction on φ_c along which the tangential stress t_{23}^*, respectively $t_{\varphi\varphi}^*$, are maximum, with $t_{23}^*(\varphi_c) = \max_{\varphi\in[-\pi,\pi]} t_{23}^*(\varphi),\ t_{\varphi\varphi}^*(\varphi_c,\beta) = \max_{\varphi\in[-\pi,\pi]} t_{\varphi\varphi}^*(\varphi,\beta)$, (see Table 1).

Table 1: SED and MTS criteria

Paper	Strain energy density Tangential stress
Craciun et al. ([10])	$s_{III}(\varphi) = (\cos^2\varphi + q^2\sin^2\varphi)^{-1/2}$ $t_{23}^* = \operatorname{Re}(\cos\varphi + \nu_3\sin\varphi)^{-1/2}$
Craciun et al. ([11])	$s_I(\varphi) = \operatorname{Re}\left[\frac{\nu_1\nu_2}{\nu_2-\nu_1}\left(\frac{\nu_1}{\chi_1(\varphi)} - \frac{\nu_2}{\chi_2(\varphi)}\right)\right]\operatorname{Re}\left[\frac{1}{\nu_2-\nu_1}\left(\frac{\nu_2 b_1}{\chi_1(\varphi)} - \frac{\nu_1 b_2}{\chi_2(\varphi)}\right)\right]-$ $\operatorname{Re}\left[\frac{\nu_1\nu_2}{\nu_2-\nu_1}\left(\frac{1}{\chi_1(\varphi)} - \frac{1}{\chi_2(\varphi)}\right)\right]\operatorname{Re}\left[\frac{\nu_1\nu_2}{\nu_2-\nu_1}\left(\frac{b_1}{\chi_1(\varphi)} - \frac{b_2}{\chi_2(\varphi)}\right)\right]-$ $\operatorname{Re}\left[\frac{\nu_1\nu_2}{\nu_2-\nu_1}\left(\frac{1}{\chi_1(\varphi)} - \frac{1}{\chi_2(\varphi)}\right)\right]\operatorname{Re}\left[\frac{\nu_1\nu_2}{\nu_2-\nu_1}\left(\frac{c_1\nu_2}{\chi_1(\varphi)} - \frac{c_2\nu_1}{\chi_2(\varphi)}\right)\right]+$ $\operatorname{Re}\left[\frac{1}{\nu_2-\nu_1}\left(\frac{\nu_2}{\chi_1(\varphi)} - \frac{\nu_1}{\chi_2(\varphi)}\right)\right]\operatorname{Re}\left[\frac{1}{\nu_2-\nu_1}\left(\frac{c_1}{\chi_1(\varphi)} - \frac{c_2}{\chi_2(\varphi)}\right)\right]$ $t_{\varphi\varphi}(\varphi) = \operatorname{Re}\left[\frac{\nu_1\nu_2}{\nu_2-\nu_1}\left(\frac{\nu_1}{\chi_1(\varphi)} - \frac{\nu_2}{\chi_2(\varphi)}\right)\right]\sin^2\varphi +$ $2\operatorname{Re}\left[\frac{\nu_1\nu_2}{\nu_2-\nu_1}\left(\frac{1}{\chi_1(\varphi)} - \frac{1}{\chi_2(\varphi)}\right)\right]\sin\varphi\cos\varphi +$ $\operatorname{Re}\left[\frac{1}{\nu_2-\nu_1}\left(\frac{\nu_2}{\chi_1(\varphi)} - \frac{\nu_1}{\chi_2(t)}\right)\right]\cos^2\varphi$
Baesu et al. ([12])	$t_{\varphi\varphi}^*(\varphi,\beta) = t_{11}^*\sin^2\varphi - (t_{12}^* + t_{21}^*)\sin\varphi\cos\varphi + t_{22}^*\cos^2\varphi$

2.2.2 Crack propagation in Femur, Tibia, and Ilium bones

In biomechanics studies regarding fracture of bones, modeled as elastic composites were done by several authors in excellent books and papers, (see [1]-[12]). Extending SED and MTS fracture criteria, (see [13], [16]-[17]) determined the crack path propagation direction and the value of the critical stress which produces crack propagation in human bones as the Tibia, Femur, and Ilium, considered to be modeled as orthotropic materials characterized by the mechanical constants as in Table 2.

Table 2: Elastic constants for Femur, Tibia, and Ilium bones, ([2], [10]-[12])

	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{21}	ν_{31}	ν_{32}	ν_{23}
Femur	12 GPa	13.4 GPa	20 GPa	4.53 GPa	5.61 GPa	6.23 GPa	0.37	0.22	0.42	0.37	0.35	0.22
Tibia	6.91 GPa	8.51 GPa	18.4 GPa	2.41 GPa	3.56 GPa	4.91 GPa	0.49	0.12	0.62	0.32	0.31	0.14
Ilium	11.6 GPa	12.2 GPa	19.9 GPa	4 GPa	5 GPa	5.4 GPa	0.42	0.23	0.44	0.39	0.38	0.23

Craciun et al. [10], studied the crack propagation in the anti-plane mode, mode III of classical fracture, in Femur and Tibia bones. Using Sih's SED generalized criterion, for finding the anti-plane crack propagation angle φ_c, was necessary to get the minimum of $s_{III}(\varphi)$, given in Table 1. Was obtained that $\varphi_c = 0$, and the same result was found using MTS generalized criterion for the shear stress t_{23}. An interesting fact is that both SED and MTS generalized criteria indicate the same value of φ_c in the vicinity of 0^o, *ie.* crack will propagate along its line.

Craciun et al. [11] and Baesu et al. [12] studied the plane crack propagation in Ilium bones subjected to mode I and respectively for mixed mode (I+II) of classical fracture.

For the crack in mode I, computing $s_I(\varphi)$, $t_{\varphi\varphi}(\varphi)$, (see Table 1), and using SED and MTS generalized criteria was obtained that the crack propagates along its line.

For the mixed mode (I+II) computing $t^*_{\varphi\varphi}(\varphi, \beta)$, (see Table 1), and using MTS generalized criteria was obtained an interesting result that, meanwhile the inclination angle β, increases from 0^o, to 90^o, the crack propagation angle φ_c decreases to 0^o, well-known result for a crack subjected in mode I of classical fracture.

Table 3: Crack propagation angle

Paper	Bone	Crack propagation angle (φ_c)
Craciun et al. ([10])	Femur	0^o
	Tibia	0^o
Craciun et al. ([11])	Ilium	0^o
Baesu et al. ([12])	Ilium	$\varphi_c = \varphi_c(\beta)$

3 Conclusions

The occurrence of cracks or other defects in bones represents an important problem that can be studied by analytical, numerical, and experimental methods. The applications of fracture mechanics mathematical models in bone mechanics represent a great success in a deeper understanding of bone fracture phenomena in medicine. Also, the interesting problems proposed by biology and medicine helped in developing new and generalized crack propagation criteria for applications of fracture mechanics.

Assuming that bones have almost the same structure as orthotropic elastic composites, using the representing theories by complex potentials of the stresses and displacements fields, their asymptotical values in the vicinity of the crack tips and extending classical crack propagation criteria from isotropic case crack propagation studies in human bones were presented.

Interesting results were obtained that in Femur, Tibia, and Ilium bones the crack propagates along its line in mode I and antiplane mode, and in mixed mode, the crack propagation direction depends on the inclination angle.

Future researches in bone biomechanics are necessary and will probably use more complex mathematical models to study the dependence of crack propagation and critical values of stresses which produces crack initialization by other factors from medicine such as bone density, healing, aging, and diseases such as diabetes and osteoporosis.

The authors declare that all subjects gave their informed consent for inclusion before they participated in the study.

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