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Hypergroups associated with HX-groups

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Abstract

We have already studied a correspondence between HX-groups and hypergroups. Now we extend this correspondence to semigroups of subsets of a group G (and to begin, we considered $\mathbf{Z}_2 \times \mathbf{Z}_3$ as group G). Moreover these subsets can have non empty intersection, they can be contained, one in another one. One calculates the fuzzy grade and we find always $\partial H = 1$.

1 Introduction

In the classic theory of HX-groups, one supposes that the structure from which one starts, is a group G, which is a subgroup of the semigroup $\mathcal{P}^*(K)$ of nonempty subsets of a group K, see for instance [3], [20], [21], [22], [23], [33], [34] and often one supposes also that

$$\forall (A,B) \in \mathcal{P}^*(K) \times \mathcal{P}^*(K),\ A \neq B \Rightarrow A \cap B = \emptyset. \tag{1}$$

In this paper one has not done always these hypotheses.

In a precedent paper [8] one considered the HX-groups associated with $\mathbf{Z}/n\mathbf{Z}$ for $n \in \{8, 9, 12, 15, 16\}$.

In this paper, we calculate those associated with $\mathbf{Z}_2 \times \mathbf{Z}_3$, without supposing always satisfied the hypothesis (1). In this new situation, we show that the fuzzy grade is always equal to one.

Let us see what means a "fuzzy grade".

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Let $(H_0, \circ)$ be a hypergroupoid.
Set $\forall (x, y) \in H^2,\ \forall u \in x \circ y$,

$$m_{x,y}(u) = 1/|x \circ y|, \quad A_1(u) = \sum_{(x,y \in H^2} m_{x,y}(u)$$

$$Q_1(u) = \{(a,b) \mid u \in a \circ b\}, \quad q_1(U) = |Q_1(U)| \ \text{ and } \ \mu_1(u) = A_1(u)/q_1(u).$$

Set now $\forall (x, y) \in H^2$,

$$x \circ_1 y = \{z \mid min\{\mu_1(x), \mu_1(y)\} \leq \mu_1(z) \leq max\{\mu_1(x), \mu_1(y)\}\}$$

and set denote by H_1 the hypergroupoid $(H_0, \circ_1)$ (which is a join space, see [4]). One goes on in the same way and from H_1 one obtains A_2, q_2, μ_2 defined similarly as A_1, q_1, μ_1, but starting from H_1. One obtains another join space $(H_2, \circ_2)$ and so on. The *fuzzy grade* of H_0, denoted by ∂H_0 is the least k, such that H_k is isomorphic to H_{k+1}.

2 HX-groups on finite groups

If $(G, \cdot)$ is a group and $\mathcal{P}^*(G)$ is the set of all non empty subsets of G then an *HX-group* is a nonempty subset H of $\mathcal{P}^*(G)$, which is a group, with respect to the hyperoperation (see [3], [20], [22], [21], [23], [33], [34]):

$$\forall (A, B) \in \mathcal{P}^*(G) \times \mathcal{P}^*(G),\ A \cdot B = \{xy \mid x \in A,\ y \in B\}.$$

Let us consider $A_0 = \mathbf{Z}_2 \times \mathbf{Z}_3 = \{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2)\}$. Set

$$B_0 = \{(0,0), (1,0)\}, \quad B_1 = \{(0,0), (0,1), (0,2)\}, \quad B_2 = \{(0,0), (0,2)\},$$

$$B_3 = \{(0,1), (1,1)\}, \quad B_4 = \{(0,2), (1,2)\}, \quad B_5 = \{(1,0), (1,1), (1,2)\}.$$

We find

$B_{1,5}$	B_1	B_5
B_1	B_1	B_5
B_5	B_5	B_1

$B_{0,3,4}$	B_0	B_3	B_4
B_0	B_0	B_3	B_4
B_3		B_4	B_0
B_4			B_3

From $B_{1,5}$ we obtain:

$B_{1,5}$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$
$(0,0)$	B_1	B_1	B_1	B_5	B_5	B_5
$(0,1)$		B_1	B_1	B_5	B_5	B_5
$(0,2)$			B_1	B_5	B_5	B_5
$(1,0)$				B_1	B_1	B_1
$(1,1)$					B_1	B_1
$(1,2)$						B_1

$B_6 = \{(0,0),(1,0),(0,2),(1,2)\}$, $B_7 = \{(0,1),(0,2),(1,1),(1,2)\}$,
$B_8 = \{(0,0),(0,1),(1,0),(1,1)\}$.
$B_{1,5}$ can be written also

$H^0_{1,5}$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$
$(0,0)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0))$ $(0,1)$ $(0,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $((1,1)$ $(1,2)$
$(0,1)$		$(0,0)$ $(0,1)$ $(0,2)$	$(0,0))$ $(0,1)$ $(0,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $((1,1)$ $(1,2)$
$(0,2)$			$(0,0)$ $(0,1)$ $(0,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $((1,1)$ $(1,2)$
$(1,0)$				$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(1,2)$	$(0,0)$ $((0,1)$ $(1,2)$
$(1,1)$					$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $((0,1)$ $(1,2)$
$(1,2)$						$(0,0)$ $((0,1)$ $(0,2)$

We deduce $A_1(0,0) = A_1(0,1) = A_1(0,2) = 29/3$,
$q_1(0,0) = q_1(0,1) = q_1(0,2) = 18$,
$\mu_1(0,0) = \mu_1(0,1) = \mu_1(0,2) = 1/3$.
$A_1(1,0) = A_1(1,1) = A_1(1,2) = 18/3$,
$q_1(1,0) = q_1(1,1) = q_1(1,2) = 18$,
$\mu_1(1,0) = \mu_1(1,1) = \mu_1(1,2) = 1/3$.
It follows that $\partial H^0_{1,5} = 1$.

$B_{2,1,5}$	B_2	B_1	B_5
B_2	B_1	B_1	B_5
B_1	B_1	B_1	B_5
B_5	B_5	B_5	B_1

One obtains $A_1(0,0) = A_1(0,1) = A_1(0,2) = 25/3 + 9/3 = 34/3$,
$A_1(1,0) = A_1(1,1) = A_1(1,2) = 30/3$.
$q_1(0,0) = q_1(0,1) = q_1(0,2) = 34,\ q_1(1,0) = q_1(1,1) = q_1(1,2) = 30$.

$H_{2,1,5}$	$(0,0)$	$(0,2)$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$
$(0,0)$	$(0,0)$ $(0,2)$ $(0,1)$	$(0,0)$ $(0,2)$ $(0,1)$	$(0,0))$ $(0,2)$ $(0,1)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(1,0)$ $((1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$
$(0,2)$	$(0,0)$ $(0,2)$ $(0,1)$	$(0,0)$ $(0,2)$ $(0,1)$	$(0,0))$ $(0,2)$ $(0,1)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(1,0)$ $((1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$
$(0,0)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0))$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(1,0)$ $((1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$
$(0,1)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0))$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(1,0)$ $((1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$
$(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0))$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(1,0)$ $((1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$
$(1,0)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0))$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(0,0)$ $((0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$
$(1,1)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0))$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(0,0)$ $((0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$
$(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0))$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(1,0)$ $(1,1)$ $(1,2)$	$(0,0)$ $((0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$	$(0,0)$ $(0,1)$ $(0,2)$

By consequence, $\mu_1(i,j) = 1/3,\ \forall(i,j)$ so $H_1 = T$ (the total hypergroup), whence $\partial(H_{2,1,5}) = 1$.

From $B_{0,3,4}$ we obtain

$H^1_{0,3,4}$	$(0,0)$	$(1,0)$	$(0,1)$	$(0,2)$	$(1,1)$	$(1,2)$
$(0,0)$	B_0	B_0	B_3	B_4	B_3	B_4
$(1,0)$		B_0	B_3	B_4	B_3	B_4
$(0,1)$			B_4	B_0	B_4	B_0
$(0,2)$				B_3	B_0	B_3
$(1,1)$					B_4	B_0
$(1,2)$						B_3

$H^1_{0,3,4}$ can be written also in the following way:

$H^0_{0,3,4}$	$(0,0)$	$(1,0)$	$(0,1)$	$(0,2)$	$(1,1)$	$(1,2)$
$(0,0)$	$(0,0)$ $(1,0)$	$(0,0)$ $(1,0)$	$(0,1))$ $(1,1)$	$(1,2)$ $(0,2)$	$(0,1)$ $(1,1)$	$(1,2)$ $((0,2)$
$(1,0)$		$(0,0)$ $(1,0)$	$(0,1))$ $(1,1)$	$(1,2)$ $(0,2)$	$(0,1)$ $(1,1)$	$(1,2)$ $((0,2)$
$(0,1)$			$(1,2)$ $(0,2)$	$(0,0)$ $(1,0)$	$(1,2)$ $(0,2)$	$(0,0)$ $(1,0)$
$(0,2)$				$(0,1)$ $(1,1)$	$(0,0)$ $(1,0)$	$(0,1)$ $(1,1)$
$(1,1)$					$(0,2)$ $(1,2)$	$(0,0)$ $(1,0)$
$(1,2)$						$(0,1)$ $(1,1)$

$A_1(0,0) = 2/2 + 8/2 = A_1(1,0) = 5$,
$A_1(0,1) = 2/2 + 10/2 = A_1(1,1) = 6$,
$A_1(0,2) = 2/2 + 10/2 = A_1(1,2) = 6$.

We obtain $q_1(0,0) = q_1(1,0) = 10$, $q_1(0,1) = q_1(1,1) = q_1(0,2) = q_1(1,2) = 12$, whence

$\mu_1(0,0) = 1/2$, $\mu_1(1,0) = 1/2$

$\mu_1(0,1) = \mu_1(1,1) = \mu_1(0,2) = \mu_1(1,2) = 1/2$. Therefore, H^1 is total, whence $\partial(H^0_{0,3,4}) = 1$.

One considers now the semigroup constructed on the set

$\{B_i \mid 0 \leq i \leq 8\} \cup \{A_0\}$.

$B_1 = \{(0,0),(0,1),(0,2)\}$,
$B_2 = \{(0,0),(0,2)\}$,
$B_5 = \{(1,0),(1,1),(1,2)\}$,
$B_0 = \{(0,0),(1,0)\}$,
$B_3 = \{(0,1),(1,1)\}$,

$B_4 = \{(0,2),(1,2\},$
$B_6 = \{(0,0),(1,0),(0,2),(1,2)\},$
$B_7 = \{(0,1),(0,2),(1,1),(1,2)\},$
$B_8 = \{(0,0),(0,1),(1,0),(1,1)\}.$
We have
$B_1B_6 = \{(0,0),(0,1),(0,2)\} \cdot \{(0,0),(1,0),(0,2),(1,2)\} =$
$\{(0,0),(1,0),(0,2),(1,1) \mid (0,1),(1,1),(0,0),(1,0)\} =$
$\{(0,2),(1,2),(0,1),(1,1)\} = A_0$
$B_1B_7 = \{(0,0),(0,1),(0,2)\} \cdot \{(0,1),(0,2),(1,1),(1,2)\} =$
$\{(0,1),(0,2),(1,1),(1,2) \mid (0,2),(0,0),(1,2),(1,0)\} = A_0$
$B_1B_8 = \{(0,0),(0,1),(0,2)\} \cdot \{(0,0),(0,1),(1,0),(1,1)\} =$
$\{(0,0),(0,1),(1,0),(1,1) \mid (0,1),(0,2),(1,1),(1,2)\} = A_0$
$B_2B_5 = \{(0,0),(0,2)\} \cdot \{(1,0),(1,1)\} =$
$\{(1,0),(1,1) \mid (1,2),(1,0)\} = B_5$
$B_2B_0 = \{(0,0),(0,2)\} \cdot \{(0,0),(1,0)\} =$
$\{(0,0),(1,0) \mid (0,2),(1,2)\} = B_6$
$B_2B_3 = \{(0,0),(0,2)\} \cdot \{(0,1),(1,1)\} =$
$\{(0,1),(1,1) \mid (0,0),(1,0)\} = B_8$
$B_2B_4 = \{(0,0),(0,2)\} \cdot \{(0,2),(1,2)\} =$
$\{(0,2),(1,2) \mid (0,1),(1,1)\} = B_7$
$B_2B_6 = \{(0,0),(0,2)\} \cdot \{(0,0),(1,0),(0,2),(1,2)\} =$
$\{(0,0),(1,0),(0,2),(1,2) \mid (0,2),(1,2),(0,1),(1,1)\} =$
$\{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2)\} = A_0$
$B_2B_7 = \{(0,0),(0,2)\} \cdot \{(0,1),(0,2),(1,1),(1,2)\} =$
$\{(0,1),(0,2),(1,1),(1,2) \mid (0,0),(0,1),(1,0),(1,1)\} = A_0$
$B_2B_8 = \{(0,0),(0,2)\} \cdot \{(0,0),(0,1),(1,0),(1,1)\} =$
$\{(0,0),(0,1),(1,0),(1,1) \mid (0,2),(0,0),(1,2),(1,0)\} = A_0$
$B_5B_0 = \{(1,0),(1,1),(1,2)\} \cdot \{(0,0),(1,0)\} =$
$\{1,0),(1,1),(1,2) \mid (0,0),(0,1),(0,2)\} = A_0$
$B_5B_3 = \{(1,0),(1,1),(1,2)\} \cdot \{(0,1),(1,1)\} =$
$\{1,1),(0,1) \mid (1,2),(0,2) \mid (1,0),(0,0)\} = A_0$
$B_5B_4 = \{(1,0),(1,1),(1,2)\} \cdot \{(0,2),(1,2)\}$
$\{(1,2),(0,2) \mid (1,0),(0,0),(1,1),(0,1)\} = A_0$
Since $B_4 \subseteq B_6$, it follows that $B_5B_6 = A_0$.
Since $B_3 \subseteq B_7$, it follows that $B_5B_7 = A_0$.
Since $B_0 \subseteq B_8$, it follows that $B_5B_8 = A_0$.
$B_0B_0 = \{(0,0),(1,0)\} \cdot \{(0,0),(1,0)\}$
$\{(0,0),(1,0) \mid (1,0),(0,0)\} = B_0$
$B_0B_3 = \{(0,0),(1,0)\} \cdot \{(0,1),(1,1)\}$
$\{(0,1),(1,1) \mid (1,1),(0,1)\} = \{(0,1),(1,1)\} = B_3$
$B_0B_4 = \{(0,0),(1,0)\} \cdot \{(0,2),(1,2)\}$

$\{(0,2),(1,2) \mid (1,2),(0,2)\} = \{(0,2),(1,2)\} = B_4$
$B_0B_5 = \{(0,0),(1,0)\} \cdot \{(1,0),(1,1),(1,2)\}$
$\{(1,0),(1,1),(1,2) \mid (0,0),(0,1),(0,2)\} = A_0$
$B_0B_6 = \{(0,0),(1,0)\} \cdot \{(0,0),(1,0),(0,2),(1,2)\}$
$\{(0,0),(1,0),(0,2),(1,2) \mid (1,0),(0,0),(1,2),(0,2)\} = B_6$
$B_0B_7 = \{(0,0),(1,0)\} \cdot \{(0,1),(0,2),(1,1),(1,2)\} =$
$\{(0,1),(0,2),(1,1),(1,2) \mid (1,1),(1,2),(0,1),(0,2)\} = B_7$
$B_0B_8 = \{(0,0),(1,0)\} \cdot \{(0,0),(0,1),(1,0),(1,1)\} =$
$\{(0,0),(0,1),(1,0),(1,1) \mid (1,0),(1,1),(0,0),(0,1)\} = B_8$
$B_3B_6 = \{(0,1),(1,1)\} \cdot \{(0,0),(1,0),(0,2),(1,2\} =$
$\{(0,1),(1,1),(1,1),(0,1) \mid (0,0),(1,0),(1,0),(0,0)\} = B_8$
$B_3B_7 = \{(0,1),(1,1)\} \cdot \{(0,1),(0,2),(1,1),(1,2\} =$
$\{(0,2),(0,0),(1,2),(1,0) \mid (1,2),(1,0),(0,2),(0,0)\} = B_6$
$B_6B_6 = \{(0,0),(1,0),(0,2),(1,2)\} \cdot \{(0,0),(1,0),(0,2),(1,2)\} =$
$\{(0,0),(1,0),(0,2),(1,2) \mid (1,0),(0,0),(1,2),(0,2)\} =$
$\{(0,2),(1,2),(0,1),(1,1) \mid (1,2),(0,2),(1,1),(0,1)\} = A_0$
$B_4B_8 = \{(0,2),(1,2)\} \cdot \{(0,0),(0,1),(1,0),(1,1)\} =$
$\{(0,2),(0,0),(1,2),(1,0) \mid (1,2),(1,0),(0,2),(0,0)\} = B_6$
$B_6B_7 = \{(0,0),(1,0),(0,2),(1,2)\} \cdot \{(0,1),(0,2),(1,1),(1,2)\} =$
$\{(0,0),(0,1),(1,0),(1,1) \mid (1,0),(1,1),(0,0),(0,1)\} = A_0$
$B_3B_8 = \{(0,1),(1,1)\} \cdot \{(0,0),(0,1),(1,0),(1,1)\} =$
$\{(0,1),(0,2),(1,1),(1,2) \mid (1,1),(1,2),(0,1),(0,2)\} = B_7$
$B_4B_6 = \{(0,2),(1,2)\} \cdot \{(0,0),(1,0),(0,2),(1,2)\} =$
$\{(0,2),(1,2),(0,1),(1,1) \mid (1,2),(0,2),(1,1),(0,1)\} = B_7$
$B_4B_7 = \{(0,2),(1,2)\} \cdot \{(0,1),(0,2),(1,1),(1,2)\} =$
$\{(0,0),(0,1),(1,0),(1,1) \mid (1,0),(1,1),(0,0),(0,1)\} = B_8$
$B_6B_8 = \{(0,0),(1,0),(0,2),(1,2)\} \cdot \{(0,0),(0,1),(1,0),(1,1)\} =$
$\{(0,0),(0,1),(1,0),(1,1) \mid (1,0),(1,1),(0,0),(0,1) \mid$
$(0,2),(0,0),(1,2),(1,0) \mid (1,2),(1,0),(0,2),(0,0)\} = A_0$
$B_7B_7 = \{(0,1),(0,2),(1,1),(1,2)\} \cdot \{(0,1),(0,2),(1,1),(1,2)\} =$
$\{(0,2),(0,0),(1,2),(1,0) \mid (0,0),(0,1),(1,0),(1,1) \mid$
$(1,2),(1,0),0,2),(0,0) \mid (1,0),(1,1),(0,0),(0,1)\} = A_0$
$B_7B_8 = \{(0,1),(0,2),(1,1),(1,2)\} \cdot \{(0,0),(0,1),(1,0),(1,1)\} =$
$\{(0,1),(0,2),(1,1),(1,2) \mid (0,2),(0,0),(1,2),(1,0) \mid$
$(1,1),(1,2),0,1),(0,2) \mid (1,2),(1,0),(0,2),(0,0)\} = A_0$
$B_8B_8 = \{(0,0),(0,1),(1,0),(1,1)\} \cdot \{(0,0),(0,1),(1,0),(1,1)\} =$
$\{(0,0),(0,1),(1,0),(1,1) \mid (0,1),(0,2),(1,1),(1,2) \mid$
$(1,0),(1,1),0,0),(0,1) \mid (1,1),(1,2),(0,1),(0,2)\} = A_0$

B^*	B_1	B_2	B_5	B_0	B_3	B_4	B_6	B_7	B_8	A_0
B_1	B_1	B_1	B_5	A_0	A_0	A_0	A_0	A_0	A_0	A_0
B_2		B_1	B_5	B_6	B_8	B_7	A_0	A_0	A_0	A_0
B_5			B_1	A_0	A_0	A_0	A_0	A_0	A_0	A_0
B_0				B_0	B_3	B_4	B_6	B_7	B_8	A_0
B_3					B_4	B_0	B_8	B_6	B_7	A_0
B_4						B_3	B_7	B_8	B_6	A_0
B_6							A_0	A_0	A_0	A_0
B_7								A_0	A_0	A_0
B_8									A_0	A_0
A_0										A_0

We shall consider now the hypergroupoid B^* on $\{B_i \mid 0 \leq i \leq 8\} \cup \{A_0\}$ and we shall study the corresponding hypergroupoid on

$H^* = \{(0,0),(0,2),(1,0),(1,2),(1,1),(0,1)\}$.

From B^* we obtain the following H^*, which we divide into two tables, H^{*1} and H^{*2} .

We first do the following notations:

denote the elements $(0,0),(0,1),(0,2)$ of B_1 by $00_{B_1}, 01_{B_1}, 02_{B_1}$, respectively;

denote the elements $(0,0),(0,2)$ of B_2 by $00_{B_2}, 02_{B_2}$, respectively;

denote the elements $(1,0),(1,1),(1,2)$ of B_5 by $10_{B_5}, 11_{B_5}, 12_{B_5}$, respectively;

denote the elements $(0,0),(1,0)$ of B_0 by $00_{B_0}, 10_{B_0}$, respectively;

denote the elements $(0,1),(1,1)$ of B_3 by $01_{B_3}, 11_{B_3}$, respectively;

denote the elements $(0,2),(1,2)$ of B_4 by $02_{B_4}, 12_{B_4}$, respectively;

denote the elements $(0,0),(1,0),(0,2),(1,2)$ of B_6 by $00_{B_6}, 10_{B_6}, 02_{B_6}, 12_{B_6}$, respectively;

denote the elements $(0,1),(0,2),(1,1),(1,2)$ of B_7 by $01_{B_7}, 02_{B_7}, 11_{B_7}, 12_{B_7}$, respectively and

H^{*1}	00_{B_1}	01_{B_1}	02_{B_1}	00_{B_2}	02_{B_2}	10_{B_5}	11_{B_5}	12_{B_5}	00_{B_0}	10_{B_0}	01_{B_3}	11_{B_3}	02_{B_4}	12_{B_4}
00_{B_1}	B_1	B_1	B_1	B_1	B_1	B_5	B_5	B_5	A_0	A_0	A_0	A_0	A_0	A_0
01_{B_1}	B_1	B_1	B_1	B_1	B_1	B_5	B_5	B_5	A_0	A_0	A_0	A_0	A_0	A_0
02_{B_1}	B_1	B_1	B_1	B_1	B_1	B_5	B_5	B_5	A_0	A_0	A_0	A_0	A_0	A_0
00_{B_2}	B_1	B_1	B_1	B_1	B_1	B_5	B_5	B_5	B_6	B_6	B_8	B_8	B_7	B_7
02_{B_2}	B_1	B_1	B_1	B_1	B_1	B_5	B_5	B_5	B_6	B_6	B_8	B_8	B_7	B_7
10_{B_5}	B_5	B_5	B_5	B_5	B_5	B_1	B_1	B_1	A_0	A_0	A_0	A_0	A_0	A_0
11_{B_5}	B_5	B_5	B_5	B_5	B_5	B_1	B_1	B_1	A_0	A_0	A_0	A_0	A_0	A_0
12_{B_5}	B_5	B_5	B_5	B_5	B_5	B_1	B_1	B_1	A_0	A_0	A_0	A_0	A_0	A_0
00_{B_0}	A_0	A_0	A_0	B_6	B_6	A_0	A_0	A_0	B_0	B_0	B_4	B_4	B_6	B_6
10_{B_0}	A_0	A_0	A_0	B_6	B_6	A_0	A_0	A_0	B_0	B_0	B_4	B_4	B_6	B_6
01_{B_3}	A_0	A_0	A_0	B_8	B_8	A_0	A_0	A_0	B_4	B_4	B_4	B_4	B_0	B_0
11_{B_3}	A_0	A_0	A_0	B_8	B_8	A_0	A_0	A_0	B_4	B_4	B_4	B_4	B_0	B_0
02_{B_4}	A_0	A_0	A_0	B_7	B_7	A_0	A_0	A_0	B_6	B_6	B_0	B_0	B_3	B_3
12_{B_4}	A_0	A_0	A_0	B_7	B_7	A_0	A_0	A_0	B_6	B_6	B_0	B_0	B_3	B_3
00_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_6	B_6	B_8	B_8	B_7	B_7
10_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_6	B_6	B_8	B_8	B_7	B_7
02_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_6	B_6	B_8	B_8	B_7	B_7
12_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_6	B_6	B_8	B_8	B_7	B_7
01_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_7	B_7	B_6	B_6	B_8	B_8
02_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_7	B_7	B_6	B_6	B_8	B_8
11_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_7	B_7	B_6	B_6	B_8	B_8
12_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_7	B_7	B_6	B_6	B_8	B_8
00_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_8	B_8	B_7	B_7	B_6	B_6
01_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_8	B_8	B_7	B_7	B_6	B_6
10_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_8	B_8	B_7	B_7	B_6	B_6
11_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_8	B_8	B_7	B_7	B_6	B_6
A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0

H^{*2}	00_{B_6}	10_{B_6}	02_{B_6}	12_{B_6}	01_{B_7}	02_{B_7}	11_{B_7}	12_{B_7}	00_{B_8}	01_{B_8}	10_{B_8}	11_{B_8}	A_0
00_{B_1}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
01_{B_1}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
02_{B_1}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
00_{B_2}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
02_{B_2}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
10_{B_5}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
11_{B_5}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
12_{B_5}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
00_{B_0}	B_6	B_6	B_6	B_6	B_7	B_7	B_7	B_7	B_8	B_8	B_8	B_8	A_0
10_{B_0}	B_6	B_6	B_6	B_6	B_7	B_7	B_7	B_7	B_8	B_8	B_8	B_8	A_0
01_{B_3}	B_8	B_8	B_8	B_8	B_6	B_6	B_6	B_6	B_7	B_7	B_7	B_7	A_0
11_{B_3}	B_8	B_8	B_8	B_8	B_6	B_6	B_6	B_6	B_7	B_7	B_7	B_7	A_0
02_{B_4}	B_7	B_7	B_7	B_7	B_8	B_8	B_8	B_8	B_6	B_6	B_6	B_6	A_0
12_{B_4}	B_7	B_7	B_7	B_7	B_8	B_8	B_8	B_8	B_6	B_6	B_6	B_6	A_0
00_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
10_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
02_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
12_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
01_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
02_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
11_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
12_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
00_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
01_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
10_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
11_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0

Let us calculate now the $\mu_1(i,j)$ for $i \in \{0,1\},\ j \in \{0,1,2\}$.

We have

$A_1(0,1) = 25/3+18\cdot3\cdot2/6+2\cdot4/4+2\cdot4/4+12\cdot2\cdot2/6+9/3+18\cdot3\cdot2/6+2\cdot 8/4+2\cdot8/4+2\cdot8/4+2\cdot8/4+4/2+2\cdot8/4+2\cdot8/4+144/6 = 101+25/3 = 328/3$, which corresponds to the sets:

B_1B_1, B_2B_2, B_1B_2 divided by $|B_1|$;

$B_1(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;

B_2B_3 divided by $|B_8|$;

B_2B_4 divided by $|B_7|$;

$B_2(B_6, B_7, B_8)$ divided by $|A_0|$;

B_5B_5 divided by $|B_1|$;

$B_1(B_0, B_3, B_4, B_5, B_6, B_7, B_8)$ divided by $|A_0|$;

B_0B_7 divided by $|B_7|$;

B_0B_8 divided by $|B_8|$;

B_4B_6 divided by $|B_7|$;

B_4B_7 divided by $|B_8|$;

B_4B_4 divided by $|B_3|$;
B_3B_6 divided by $|B_8|$;
B_3B_8 divided by $|B_7|$;
and 144 divided by $|A_0|$, respectively. So, $A_1(0,1) = 328/3$.
$q_1(0,1) = 25+108+8+8+48+9+108+16+16+16+16+4+16+16+144 = = 558$;
$\mu_1(0,1) = 328/(3\cdot 558) = 0.1959$.
$A_1(1,1) = 2\cdot 9/3+18\cdot 3\cdot 2/6+2\cdot 2\cdot 3/3+2\cdot 4/4+2\cdot 4/4+12\cdot 2\cdot 2/6+9/3+ 18\cdot 3\cdot 2/6+4/2+2\cdot 8/4+2\cdot 8/4+2\cdot 8/4+2\cdot 8/4+2\cdot 8/4+2\cdot 8/4+144/6 = 6+4+18+2+2+8+3+18+4+4+4+4+2+4+4+24 = 111$, which corresponds to the sets:
B_1B_5 divided by $|B_5|$;
$B_1(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;
B_2B_5 divided by $|B_5|$;
B_2B_3 divided by $|B_8|$;
B_2B_4 divided by $|B_7|$;
$B_2(B_6, B_7, B_8)$ divided by $|A_0|$;
B_5B_5 divided by $|B_1|$;
$B_1(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;
B_4B_4 divided by $|B_3|$;
B_3B_6 divided by $|B_8|$;
B_3B_8 divided by $|B_7|$;
B_4B_6 divided by $|B_7|$;
B_4B_7 divided by $|B_8|$;
B_0B_7 divided by $|B_7|$;
B_0B_8 divided by $|B_8|$;
and 144 divided by $|A_0|$, respectively. So, $A_1(1,1) = 111$.
$q(1,1) = 18+12+108+8+8+48+9+108+16+16+16+16+4+16+16+144 = 563$
$\mu_1(1,1) = 111/563 = 0.19716$.
$A_1(1,0) = 2\cdot 9/3+18\cdot 3\cdot 2/6+2\cdot 3\cdot 2/3+2\cdot 4/4+2\cdot 4/4+12\cdot 2\cdot 2/6+18\cdot 3\cdot 2/6+ 4/2+2\cdot 4/4+2\cdot 8/4+2\cdot 8/4+2\cdot 4/2+2\cdot 8/4+2\cdot 8/4+2\cdot 8/4+2\cdot 8/4+144/6 = 114$, which corresponds to the sets:
B_1B_5 divided by $|B_5|$;
$B_1(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;
B_2B_5 divided by $|B_5|$;
B_2B_0 divided by $|B_6|$;
B_2B_3 divided by $|B_8|$;
$B_2(B_6, B_7, B_8)$ divided by $|A_0|$;
$B_5(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;
B_0B_0 divided by $|B_0|$;

B_0B_4 divided by $|B_6|$;
B_0B_6 divided by $|B_6|$;
B_0B_8 divided by $|B_8|$;
B_3B_4 divided by $|B_0|$;
B_3B_6 divided by $|B_8|$;
B_3B_7 divided by $|B_6|$;
B_4B_7 divided by $|B_8|$;
B_4B_8 divided by $|B_6|$
and 144 divided by $|A_0|$, respectively. So, $A_1(1,0) = 114$.

$q_1(1,0) = 18 + 108 + 12 + 8 + 8 + 48 + 108 + 4 + 8 + 16 + 16 + 16 + 8 + 16 + 16 + 16 + 144 = 570$;

$\mu_1(1,0) = 114/570 = 0.2$.

$A_1(1,2) = 2{\cdot}9/3+18{\cdot}3{\cdot}2/6+2{\cdot}3{\cdot}2/3+2{\cdot}4/4+2{\cdot}4/4+12{\cdot}2{\cdot}2/6+18{\cdot}3{\cdot}2/6+2{\cdot}4/2+2{\cdot}4/4+2{\cdot}8/4+2{\cdot}8/4+2{\cdot}8/4+2{\cdot}8/4+2{\cdot}8/4+2{\cdot}8/4++4/2+144/6 = 114$, which corresponds to the sets:

B_1B_5 divided by $|B_5$;
$B_1(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;
B_2B_5 divided by $|B_5|$;
B_2B_0 divided by $|B_6|$;
B_2B_4 divided by $|B_7|$;
$B_2(B_6, B_7, B_8)$ divided by $|A_0|$;
$B_5(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;
B_0B_3 divided by $|B_4|$;
B_0B_4 divided by $|B_6|$;
B_0B_6 divided by $|B_6|$;
B_0B_7 divided by $|B_7|$;
B_4B_6 divided by $|B_7|$;
B_4B_8 divided by $|B_6|$;
B_3B_7 divided by $|B_6|$;
B_3B_8 divided by $|B_7|$;
B_3B_3 divided by $|B_4|$
and 144 divided by $|A_0|$, respectively. So, $A_1(1,2) = 114$.

$q_1(1,2) = 18 + 108 + 12 + 8 + 8 + 48 + 108 + 8 + 8 + 16 + 16 + 16 + 16 + 16 + 16 + 4 + 144 = 570$;

$\mu_1(1,2) = 114/570 = 0.2$.

$A_1(0,0) = 25/3+18\cdot2\cdot3/6+4\cdot2/4+4\cdot2/4+12\cdot2\cdot2/6+9/3+18\cdot3\cdot2/6+4/2+2\cdot4/4+2\cdot8/4+2\cdot8/4+2\cdot4/2+2\cdot8/4+2\cdot8/4+2\cdot8/4+144/6 = 346/3$,

$q_1(0,0) = 25 + 108 + 8 + 8 + 48 + 9 + 108 + 4 + 8 + 16 + 16 + 8 + 16 + 16 + 16 + 16 + 144 = 574$;

$\mu_1(0,0) = 346/(3 \cdot 574) = 0.2009$.

$A_1(0,2) = 25/3+18\cdot 2\cdot 3/6+4\cdot 2/4+4\cdot 2/4+12\cdot 2\cdot 2/6+9/3+18\cdot 3\cdot 2/6+2\cdot 4/4+2\cdot 4/2+2\cdot 8/4+2\cdot 8/4+2\cdot 8/2+2\cdot 8/4+2\cdot 8/4+2\cdot 8/4+2\cdot 8/4+144/6 = 346/3$, which corresponds to the sets:

$(B_1 + B_2)\cdot(B_1 + B_2)$ divided by $|B_1|$;
$B_1(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;
B_2B_0 divided by $|B_6|$;
B_2B_4 divided by $|B_7|$;
$B_2(B_6, B_7, B_8)$ divided by $|A_0|$;
B_5, B_5 divided by $|B_1|$;
$B_5(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;
B_0B_4 divided by $|B_6|$;
B_0B_3 divided by $|B_4|$;
B_0B_6 divided by $|B_6|$;
B_0B_7 divided by $|B_7|$;
B_3B_7 divided by $|B_6|$;
B_3B_8 divided by $|B_7|$;
B_4B_6 divided by $|B_7|$;
B_4B_8 divided by $|B_6|$;
B_3B_3 divided by $|B_4|$
and 144 divided by$|A_0|$ respectively. So, $A_1(0,2) = 346/3$.

$q_1(0,2) = 25 + 108 + 8 + 8 + 48 + 9 + 108 + 8 + 8 + 16 + 16 + 16 + 16 + 16 + 16 + 4 + 144 = 574$;

$\mu_1(0,2) = \mu_1(0,0) = 346/(3\cdot 574) = 0.200929$.

So, we have
$\mu_1(0,1) = 0.1959$
$\mu_1(1,1) = 0.19716$
$\mu_1(1,0) = \mu_1(1,2) = 0.2$
$\mu_1(0,0) = \mu_1(0,2) = 0.200929$.

Let us denote
$(0,1)$ by 1, $(1,1)$ by 2,
$(1,0)$ by 3, $(1,2)$ by 4,
$(0,0)$ by 5, $(0,2)$ by 6.

Let H be the set $\{1,2,3,4,5,6\}$.

Hence we obtain the following structure:

H_1^*	1	2	3	4	5	6
1	1	$1,2$	$1,2,3,4$	$1,2,3,4$	H	H
2		2	$2,3,4$	$2,3,4$	$2,3,4,5,6$	$2,3,4,5,6$
3			$3,4$	$3,4$	$3,4,5,6$	$3,4,5,6$
4				$3,4$	$3,4,5,6$	$3,4,5,6$
5					$5,6$	$5,6$
6						$5,6$

From H_1^* we obtain
$A_2(2) = 1 + 2/2 + 4/4 + 4/3 + 4/5 + 4/6 = 29/5.$
$q_2(2) = 2 + 1 + 4 + 4 + 4 + 4 = 19$
$\mu_2(2) = 29/(5 \cdot 19) = 0.305263.$
$A_2(3) = A_2(4) = 4/2 + 4/4 + 4/3 + 4/5 + 8/4 + 4/6 = 39/5.$
$q_2(3) = q_2(4) = 4 + 4 + 4 + 4 + 4 + 8 = 28$
$\mu_2(3) = \mu_2(4) = 39/(5 \cdot 28) = 0.27857.$
$A_2(5) = A_2(6) = 4/2 + 8/3 + 4/5 + 4/6 = 82/15.$
$q_2(5) = q_2(6) = 4 + 8 + 4 + 4 = 20$
$\mu_2(5) = \mu_2(6) = 82/(15 \cdot 20) = 0.27333.$
$A_2(1) = 1 + 2/2 + 4/4 + 4/6 = 11/3$
$q_2(1) = 11, \quad \mu_2(1) = 11/(11 \cdot 3) = 0.3333$
Notice that
$\mu_2(1) = 0.3333 > \mu_2(2) = 0.305263 > \mu_2(3) = \mu_2(4) = 0.27857 > \mu_2(5) = \mu_2(6) = 0.2733.$
Thus, $H_2 \simeq H_1$, whence $\partial H_1^* = 1$.

3 Conclusion

Clearly, H_2 and H_1 are isomorphic and by consequence, the fuzzy grade of H^* is one.

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