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EB lifetime distributions as alternative to the EP lifetime distributions

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Abstract

In this paper we consider lifetime distributions called EB-Max distribution and EB-Min. In the conditions of the Poisson's Limit Theorem it is shown that EB-Max distribution may be approximated by its analogous called EP-Max lifetime distribution and EB-Min distribution may be approximated by its analogous EP-Min lifetime distribution. Further, as example, two methods are provided to simulate pseudo random number for EB-Min distribution and we apply EM algorithm to estimate parameters of EB-Min distribution. An example with real data is also presented and the proposed simulation algorithms where implemented in Maple.

1 Introduction

In paper [6] it was introduced EB-Min distribution compounding exponentially distributed lifetimes with zero truncated binomially distributed r.v. as alternative to the EP-Min lifetime distribution introduced by Kuş, C. [5] mixing the same lifetime with zero truncated Poisson distributed r.v.. In the both cases lifetimes are represented as minimum of k exponentially distributed r.v., $k = 1, 2, ...$ If we substitute minimum by maximum we abtain EB-Max [4,7] and EP-max [3] distributions which are a special case of complementary exponential power series (CEPS) distributions introduced in 2012 by Jose Flores D. et al. The purpose of this paper is to study possible connections between EP and EP lifetime distributions.

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2 EP and EB lifetime distributions

First of all, we write a general formula [4] for distribution of r.v. max(W_1, W_2, ... ,W_K) , where $(W_i)_{i\geq 1}$ are independent identically distributed random variables (i.i.d.r.v.) and K is a discrete r.v. such that $\mathbf{P}(K \in \{1,2,...\}) = 1$. So, we consider that distribution function (d.f.) of r.v. W_i is $F(x) = \mathbf{P}(W_i \leq x)$, $i \geq 1$. Then, due of independence of r.v. $(W_i)_{i\geq 1}$, the d.f. of r.v.$Y_k = \max(W_1,W_2 ,...,W_k)$ is

$$F_{Y_k}(x) = \mathbf{P}(Y_k \leq x) = [F(x)]^k \,,\ \forall k = 1, 2, ... \,.$$

This means that d.f. of r.v. $Y = \max(W_1, W_2 ,...,W_K)$ is a mixture of d.f. $F_{Y_k}(x)$ with respect to the distribution of r.v. K. Indeed

$$F_Y(x) = \mathbf{P}(Y \leq x) = \mathbf{P}(\max(W_1, W_2 \ ,..., W_K) \leq x) = \sum_{k\geq 1} [F(x)]^k \, \mathbf{P}(K = k).(\mathbf{1})$$

This formula show us that, if $(W_i)_{i\geq 1}$ are r.v. of absolutely continuos type, then Y is a r.v. of the same type and its probability density function (p.d.f.) is

$$f_Y(x) = F_Y^{'}(x) = \sum_{k\geq 1} kF^{'}(x)\,[F(x)]^{k-1}\,\mathbf{P}(K = k).(\mathbf{2})$$

Similarly, can be write the general formula for distribution of r.v.$Z=$ $\min(W_1,\ W_2,\ ...\ ,W_K)$. So, density function and probability density function are

$$\begin{aligned} F_Z(x) &= \mathbf{P}(Z < x) = 1 - \mathbf{P}(\min(W_1, W_2 \ ,..., W_K) \geqslant x) \\ &= 1 - (1 - \sum_{k\geq 1} [F(x)])^k \mathbf{P}(K = k).(\mathbf{3}) \end{aligned}$$

$$f_Z(x) = F_Z^{'}(x) = \sum_{k\geq 1} kF^{'}(x)\,[1 - F(x)]^{k-1}\,\mathbf{P}(K = k).\ (\mathbf{4})$$

Applying formulas $(\mathbf{1}) - (\mathbf{2})$, and considering that $(W_i)_{i\geq 1}$ are independent identically exponentially distributed r.v. with parameter λ, $\lambda > 0$, i.e.

$$F(x) = \mathbf{P}(W_i \leq x) = (1 - e^{-\lambda x}) \cdot \ I_{[0,+\infty)}(x),\ i \geq 1$$

and K is a zero truncated binomially distributed r.v., i.e.,

$$\mathbf{P}(K = k) = \frac{1}{1 - (1-p)^n} \mathbb{C}_n^k\ p^k\ (1-p)^{n-k},\ k = \overline{1.n},\ p \in (0, 1),$$

we obtain the distribution called EB-max [4,7] with d.f. of r.v. $Y = \max(W_1, W_2, ..., W_K)$ given by formula

$$U_{\max}(x) = \frac{(1 - pe^{-\lambda x})^n - (1-p)^n}{1 - (1-p)^n} \cdot \; I_{[0,+\infty)}(x) \qquad (\mathbf{5})$$

and p.d.f. of r.v. Y is

$$u_{\max}(x) = \frac{np\lambda e^{-\lambda x}}{1 - (1-p)^n}(1 - pe^{-\lambda x})^n \cdot \; I_{[0,+\infty)}(x) \;, \qquad (\mathbf{6})$$

where

$$I_{[0,+\infty)}(x) = \begin{cases} 0, \; if \; x < 0, \\ 1, \; if \; x \geq 0. \end{cases}$$

For the same lifetimes $(W_i)_{i \geq 1}$, substituting zero truncated binomial distribution for r.v. K by zero truncated Poisson distribution with parameter μ, $\mu > 0$, i.e.,

$$\mathbf{P}(K = k) = \frac{1}{1 - e^{-\mu}} \frac{\mu^k}{k!} e^{-\mu}, \; k = 1, 2, ...,$$

in [3] it was introduced another new lifetime distribution called EP-Max distribution given by formula

$$V_{\max}(x) = \frac{e^{-\mu e^{-\lambda x}} - e^{-\mu}}{1 - e^{-\mu}} \cdot \; I_{[0,+\infty)}(x). \qquad (\mathbf{7})$$

In similar way it was introduced EB-Min [6] and EP-Min [5] life time distributions. According to [6] EB-Min distribution is given by d.f.

$$U_{\min}(x) = \left\{ 1 - \frac{1}{1 - (1-p)^n} \left\{ \left[1 - p(1 - e^{-\lambda x}) \right]^n - (1-p)^n \right\} \right\} \cdot \; I_{[0,+\infty)}(x). \; (\mathbf{8})$$

and according to[5] EP-Min distribution is given by d.f.

$$V_{\min}(x) = \frac{e^{\mu e^{-\lambda x}} - e^{\mu}}{1 - e^{\mu}} \cdot I_{[0,+\infty)}(x). \; (\mathbf{9})$$

3 Approximating EB distributions by EP distributions

As we know [2], Poisson's Limit Theorem show us that, in some conditions, binomial distribution may by approximated by Poisson distribution. This fact suggest us that between d.f. $U_{\max}(x)$ and $V_{\max}(x)$ and, on the other hand, between d.f. $U_{\min}(x)$ and $V_{\min}(x)$ does exist the similar connections. Indeed, it is true the following

Proposition (Poisson's Limit Theorem for EB an EP distributions). *In the conditions of the Poisson's Limit Theorem, i.e., if* $n \longrightarrow +\infty$ *and*
$p \longrightarrow 0$ *in a such way that* $np \longrightarrow \mu$, $\mu > 0$, *then [4,7]*

$$\lim_{\substack{n \longrightarrow +\infty \\ p \longrightarrow 0}} U_{\max}(x) = \lim_{\substack{n \longrightarrow +\infty \\ p \longrightarrow 0}} \frac{(1-pe^{-\lambda x})^n - (1-p)^n}{1-(1-p)^n} \cdot I_{[0,+\infty)}(x)$$

$$= V_{\max}(x),\ \forall x \in \mathbb{R}$$

and

$$\lim_{\substack{n \longrightarrow +\infty \\ p \longrightarrow 0}} U_{\min}(x) =$$

$$\lim_{\substack{n \longrightarrow +\infty \\ p \longrightarrow 0}} \left\{1 - \frac{1}{1-(1-p)^n}\left\{\left[1-p(1-e^{-\lambda x})\right]^n - (1-p)^n\right\}\right\} \cdot I_{[0,+\infty)}(x)$$

$$= V_{\min}(x),\ \forall x \in \mathbb{R}.$$

Remark. Let us observe that the second part of this Proposition will be confirmed empirically in the Example 2 of our work (see Table 5).

4 On the statistical simulation of EB-Min distribution

We consider, as example, two techniques for statistical simulation for EB-min distribution [6,7].

The first method is based on the fact that distribution of r.v. $X \sim EB - Min(\lambda; n, p)$ coincide with distribution of r.v. $\min\limits_{1 \leqslant i \leqslant Z} W_i$, where $(W_i)_{i \geqslant 1}$ are i.i.d.r., $W_i \sim Exp(\lambda)$, $\lambda > 0$, and r.v.Z is zero truncated binomially distributed with parameters n, $n \in \{2, 3, ...\}$ and p, $p \in (0, 1)$. So, we deduce the following algorihm

Algorithm MixtEB-Min for statistical generation of number x **of r.v.** $X \sim EB - Min(\lambda; n, p)$.

Step 1. *Simulate value* z *of r.v.* $Z \sim Binomial\,(n, p)$ *until* z *become different of zero*;

Step 2. *For this* z *we simulates values* w_i, $i = 1, 2, ...z$ *as a values of* z *r.v. i.i.d.* $Exp\,(\lambda)$;

Step 3. *Take* $x = \min\limits_{1 \leq i \leq z} w_i$.

The second is enveloping procedure, as a particular case of rejection's Method, presented in [8], which conduct us to the

Algorithm EnvEB-Min for statistical generation of number x of r.v. $X \sim EB - Min(\lambda; n, p)$.

From (**8**) we deduce that p.d.f. of r.v. $X \sim EB - Min(\lambda; n, p)$ is given by formula

$$u_{\min}(x) = \frac{np\lambda e^{-\lambda x}}{1-(1-p)^n}\left[1-p(1-e^{-\lambda x})\right]^{n-1} \cdot I_{[0,+\infty)}(x).(\mathbf{10})$$

For $u_{\min}(x)$ we choose as enveloping p.d.f.

$$h\,(x) = \lambda e^{-\lambda x} \cdot I_{[0,+\infty)}(x).$$

So, for $\forall x \geq 0$,

$$r\,(x) = \frac{u_{\min}(x)}{h\,(x)} = \frac{np}{1-(1-p)^n}\left[1-p(1-e^{-\lambda x})\right]^{n-1},$$

and

$$r'\,(x) = -\frac{\lambda np^2\,(n-1)\,e^{-\lambda x}}{1-(1-p)^n}\left[1-p(1-e^{-\lambda x})\right]^{n-2},$$

We observe that $r'\,(x) < 0$, which implies $r\,(x) < r\,(0) = \alpha = \frac{np}{1-(1-p)^n}$, where $\alpha > \mathbf{1}$.

Step 1. *Generate value* u *of r.v.* $U \sim U([0,1])$;

Step 2. *If* $u < 10^{-7}$ *then go to Step 1*;

Step 3. *Generate value* u_1 *of r.v.* $U \sim U([0,1])$;

Step 4. *If* $u_1 > (1-p(1-u))^{n-1}$ *then go to Step 3*;

Step.5. *Take* $x = -\frac{1}{\lambda}\log(u)$.

The probability of acceptance in this case is

$$p_a = \frac{1}{\alpha} = \frac{1-(1-p)^n}{np}.$$

In the following Table 1 we have, as a results, CPU's time (in seconds) $\theta(n, p, \lambda)$, executing $N = 10000$ simulations in the cases of both algorithms:

Table 1. CPU's time (in seconds) $\theta(n,p,\lambda)$

(n,p,λ)	$\theta(n,p,\lambda)$(sec) for Algorithm MixtEB-Min	$\theta(n,p,\lambda)$(sec) for Algorithm EnvEB-Min
$(3,0.5,1)$	0.330	0.235
$(5,0.5,1)$	0.335	0.362
$(3,0.5,2)$	0.352	0.268
$(5,0.5,2)$	0.386	0.390
$(3,0.9,1)$	0.441	0.413
$(3,0.9,2)$	0.592	0.438
$(5,0.9,2)$	0.607	0.480
$(3,0.2,1)$	0.623	0.206
$(3,0.2,2)$	0.652	0.210
$(5,0.2,2)$	0.599	0.234
$(10,0.5,7)$	0.556	0.507
$(15,0.5,2)$	0.607	0.818

On the base of the same simulations we may see (Table 2 and Table 3) how look mean value $\mathbb{E}X$, variance $\mathbb{D}X$ and corresponding sample mean value $\bar{x}$ and variance s^2 for r.v. $X \sim EB - Min(\lambda; n, p)$ in case of each Algorithm.

Table 2. Algorithm MixtEB-Min

Case 1: $n=3$, $p=0.5$, $\lambda=1$		Case 2: $n=5$, $p=0.5$, $\lambda=1$	
$\mathbb{E}X=0.690$	$\mathbb{D}X=0.551$	$\mathbb{E}X=0.476$	$\mathbb{D}X=0.289$
$\bar{x}=0.676$	$s^2=0.577$	$\bar{x}=0.467$	$s^2=0.315$
Case 3: $n=3$, $p=0.5$, $\lambda=2$		Case 4: $n=5$, $p=0.5$, $\lambda=2$	
$\mathbb{E}X=0.345$	$\mathbb{D}X=0.137$	$\mathbb{E}X=0.238$	$\mathbb{D}X=0.072$
$\bar{x}=0.324$	$s^2=0.145$	$\bar{x}=0.234$	$s^2=0.078$

Table 3. Algorithm EnvEB-Min

Case 1: $n=3$, $p=0.5$, $\lambda=1$		Case 2: $n=5$, $p=0.5$, $\lambda=1$	
$\mathbb{E}X=0.690$	$\mathbb{D}X=0.551$	$EX=0.476$	$\mathbb{D}X=0.289$
$\bar{x}=0.669$	$s^2=0.553$	$\bar{x}=0.481$	$s^2=0.298$
Case 3: $n=3$, $p=0.5$, $\lambda=2$		Case 4: $n=5$, $p=0.5$, $\lambda=2$	
$\mathbb{E}X=0.345$	$\mathbb{D}X=0.137$	$\mathbb{E}X=0.238$	$\mathbb{D}X=0.072$
$\bar{x}=0.359$	$s^2=0.151$	$\bar{x}=0.225$	$s^2=0.069$

5 EM algorithm and its application to the $EB - Min$ distribution

If we consider sample of size m from population of r.v. X, i.e., $(x_1,\ x_2,...,\ x_m) \sim X$, where $X \sim EB - Min(\lambda; n, p)$, then, from (**10**) we deduce that maximum likelihood function $L(x_1,\ x_2,...,\ x_m; n, p, \lambda)$ is defined by formula

$$L(x_1, x_2, ..., x_m; \lambda, p) = \prod_{i=1}^{m} \frac{np\lambda \exp\{-\lambda x_i\}}{1-(1-p)^n} \left[1 - p(1-\exp\{-\lambda x_i\})\right]^{n-1} =$$

$$\frac{(np\lambda)^m \exp\{-\lambda \sum\limits_{i=1}^{m} x_i\}}{\left[1-(1-p)^n\right]^m} \prod_{i=1}^{m} \left[1 - p(1-\exp\{-\lambda x_i\})\right]^{n-1}$$

In order to obtain equations of Maximum likelihood estimations (MLE) $\widehat{\lambda}, \widehat{p}$ for parameters λ, p we consider

$$\ln L(x_1, x_2, ..., x_m; \lambda, p) = m(\ln n + \ln p + \ln \lambda) -$$

$$\lambda \sum_{i=1}^{m} x_i \ - m \ln\left[1-(1-p)^n\right] - \ (n-1) \sum_{i=1}^{m} \ln[1 - p(1-\exp\{-\lambda x_i\})].$$

Parameter n considered be known, equations of MLE are

$$\begin{cases} \frac{\partial \ln L}{\partial \lambda} = \frac{m}{\lambda} - \sum\limits_{i=1}^{m} x_i + (n-1) \sum\limits_{i=1}^{m} \frac{p x_i \exp\{-\lambda x_i\}}{1-p(1-\exp\{-\lambda x_i\})} = 0, \\ \frac{\partial \ln L}{\partial p} = \frac{m}{p} - \frac{mn(1-p)^{n-1}}{1-(1-p)^n} - (n-1) \sum\limits_{i=1}^{m} \frac{\exp\{-\lambda x_i\}-1}{1-p(1-\exp\{-\lambda x_i\})} = 0. \end{cases}$$

But this equations, with respect to the parameters λ and p, it is very difficult to solve by means of existing methods. So, EM Algorithm proposed in [1], is more preferable. In this case r.v. Z it is interpreted as missing or latent variable.

Let's consider a sample $((x_1, z_1),\ (x_2, z_2),\ ...\ ,\ (x_m, z_m))$ of n observations on the r.v. (X, Z). That means $((x_1, z_1),\ (x_2, z_2),\ ...\ ,\ (x_m, z_m))$ may be interpreted as complete data and $(x_1,\ x_2,...,\ x_m)$ as incomplete data's observations.

To formulate EM algorithm we need to know conditional mean value $\mathbb{E}(Z \mid X; \theta)$, where parameter $\theta = (\lambda, p)$. P.d.f. $u_{\min}(x; \theta)$ of r.v. X, corresponding to the incomplete data, is given by formula (**10**). But p.d.f. of r.v. (X, Z), corresponding to the complete data, is given by formula

$$u(x, z; \theta) = \frac{z\lambda \exp\{-\lambda z x\}}{1-(1-p)^n} \mathbb{C}_n^z p^z (1-p)^{n-z} \cdot I_{[0,+\infty)}(x).$$

So, p.d.f. of r.v. Z for given (known) X, i.e., p.d.f. of r.v. Z conditioned by r.v. X, my by expressed, for $x > 0$, as

$$u(z \mid x;\theta) = u(x,z;\theta) \ / \ u_{\min}(x;\theta) =$$

$$\frac{\frac{z\lambda\exp\{-\lambda zx\}}{1-(1-p)^n}\mathbb{C}_n^k p^k(1-p)^{n-k}}{\frac{np\lambda\exp\{-\lambda x\}}{1-(1-p)^n}\left[1-p(1-\exp\{-\lambda x\})\right]^{n-1}} =$$

$$\frac{z\lambda\exp\{-\lambda(z-1)x\}\mathbb{C}_n^z p^{z-1}(1-p)^{n-z}}{np\left[1-p(1-\exp\{-\lambda x\})\right]^{n-1}}.$$

In this way, on the base of $u(z \mid x;\theta)$, we may calculate conditional mean value

$$\mathbb{E}(Z \mid X;\theta) \overset{\mathbf{def}}{=} \sum_{z=1}^{n} z\cdot u(z \mid x;\theta) =$$

$$\frac{1-p(1-n\exp\{-\lambda x\})}{1-p(1-\exp\{-\lambda x\})}. \quad (\mathbf{11})$$

Now, we may describe EM algorithm as iterative approximation of unknown parameter $\theta = (\lambda, p)$ by $\theta^{(h)} = (\lambda^{(h)}, p^{(h)})$ calculated at the same step $h \geqslant 1$ such that the condition

$$\max(\left|\lambda^{(h)} - \lambda^{(h-1)}\right|, \left|p^{(h)} - p^{(h-1)}\right|) \leqslant \varepsilon \text{ or } h = N \qquad (\mathbf{12})$$

was satisfied, where $\varepsilon > 0$ is given error and N is pre-established number of iterations.

1. Take $\lambda = \lambda^{(0)}, p = p^{(0)}$ for some $\lambda^{(0)} > 0$ and $p^{(0)} \in (0,1)$;

2. **Step E** (*Expectation*): for iteration h, $h \geqslant 1$, calculate expected values $z_i^{(h-1)}$, $i = \overline{1,m}$, according to the formula (**11**)

$$z_i^{(h-1)} = \frac{1-p^{(h-1)}(1-n\exp\{-\lambda^{(h-1)}x_i\})}{1-p^{(h-1)}(1-\exp\{-\lambda^{(h-1)}x_i\})};$$

3. **Step M** (*Maximization*): by means of MLE (*maximum likelihood estimation*) method, taking as sample size $((x_1, z_1^{h-1}), (x_2, z_2^{h-1}), \ldots, (x_m, z_m^{h-1}))$ with likelihood function

$$L(x_1, x_2, , ..., x_m, z_1^{h-1}, z_2^{h-1}, ..., z_m^{h-1}; \theta^{(h-1)}) =$$

$$\prod_{i=1}^{m}\frac{z_i^{h-1}\lambda^{(h-1)}\exp\{-\lambda^{(h-1)}z_i^{h-1}x\}}{1-(1-p^{(h-1)})^n}\mathbb{C}_n^{z_i^{h-1}}(p^{(h-1)})^{z_i^{h-1}}(1-p^{(h-1)})^{n-z_i^{h-1}},$$

find the next iteration $\theta^{(h)}$ of estimation $\overset{\wedge}{\theta}$ for parameter $\theta = (\lambda, p)$;

4. Check the condition (**12**). *If NOT, then GO TO 2, ELSE* $\overset{\wedge}{\theta} := \theta^{(h)}$, *STOP*.

Example 1. Let's consider as a statistical data observations, results of $m = 10000$ simulations of r.v. $X \sim EB - Min(\lambda; n, p)$ and results $\lambda^{(h)}, p^{(h)}$ of estimation of parameters λ, p by means of EM algorithm for different values of error ε and maximal number of iterations K.

Case a) $\lambda = 4, p = 0.25, n = 5,\ \varepsilon = 10^{-4}, K = 400$: For initial values $\lambda^{(0)} = \mathbb{E}X$, $p^{(0)} = \mathbb{D}X$ we have: $h = 400$, $\lambda^{(h)} = 4.102892$, $p^{(h)} = 0.22659$;

Case b) $\lambda = 4, p = 0.25, n = 2,\ \varepsilon = 10^{-4}, K = 500$: For initial values $\lambda^{(0)} = \mathbb{E}X$, $p^{(0)} = \mathbb{D}X$ we have: $h = 500$, $\lambda^{(h)} = 4.037867$, $p^{(h)} = 0.2724457$.

Example 2. Let's consider the data about life time intervals between two successive strong earthquakes used in [5], see Table 4:

Table 4. Earthquakes in North Anatolia fault zones

Date	*Longitude*	*Latitude*	*Magnitude (Mw)*
04.12.1905	39	39	6.8
09.02.1909	38	40	6.3
25.06.1910	34	41	6.2
24.01.1916	36.83	40.27	7.1
18.05.1929	37.9	40.2	6.1
19.04.1938	33.79	39.44	6.6
26.12.1939	39.51	39.8	7.9
30.07.1940	35.25	39.64	6.2
20.12.1940	39.2	39.11	6
08.11.1941	39.5	39.74	6
11.12.1942	34.83	40.76	6.1
20.12.1942	36.8	40.7	7
20.06.1943	30.51	40.85	6.5
26.11.1943	33.72	41.05	7.2
01.02.1944	32.69	41.41	7.2
26.10.1945	33.29	41.54	6
13.08.1951	32.87	40.88	6.9
07.09.1953	33.01	41.09	6.4
20.02.1956	30.49	39.89	6.4
26.05.1957	31	40.67	7.1
22.07.1967	30.69	40.67	7.2
03.09.1968	32.39	41.81	6.5
13.03.1992	39.63	39.72	6.1
08.03.1997	35.44	40.78	6
12.11.1999	31.21	40.74	7.2

After fitting EB-Min distribution to the above mentioned *sample of size* $m = 25$, using Kolmogorov-Smirnov (K-S) statistics we may compare our results (see the first row of Table 5) with results from [5] (see the last for rows of Table 5).

Table 5. Parameter estimates, Kolmogorov-Smirnov Statistics and p-values from the fit of the each of the 5 distributions

Distribution	*EM Estimates*	K-S statistics	$\mathbf{p} - values$
$EB - Min(\lambda; 3, p)$	$\widehat{\theta} = (\widehat{\lambda}, \widehat{p}) = (4.68 \times 10^{-4}, 0.585596)$	0.125	0.9942
$EP - Min(\lambda, \mu)$	$\widehat{\theta} = (\widehat{\lambda}, \widehat{\mu}) = (2.6377, 3.56 \times 10^{-4})$	0.0972	0.9772
$EG(\lambda, p)$	$\widehat{\theta} = (\widehat{\lambda}, \widehat{p}) = (6.995 \times 10^{-4}, 1.154 \times 10^{-5})$	0.1839	0.3914
$Weibull(\lambda, \mu)$	$\widehat{\theta} = (\widehat{\lambda}, \widehat{\mu}) = (8.12 \times 10^{-4}, 0.7854)$	0.1004	0.969
$Gamma(\lambda, \mu)$	$\widehat{\theta} = (\widehat{\lambda}, \widehat{\mu}) = (4.98 \times 10^{-4}, 0.7117)$	0.1239	0.8551

6 Conclusions

This paper shows that EB-Min distribution is a good alternative to the EP-Min distribution, even to the EG (compounding exponentially distributed lifetimes with zero truncated geometrically distributed r.v.), Weibull or Gamma distributions.

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