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On the recovery of the doping profile in an time-dependent drift-diffusion model

Bin Wu and Zewen Wang

Abstract

We consider an inverse problem arising from an time-dependent drift-diffusion model in semiconductor devices, which is formulated in terms of a system of parabolic equations for the electron and hole densities and the Poisson equation for the electric potential. This inverse problem aims to identify the doping profile from the final overdetermination data of the electric potential. By using the Schauder's fixed point theorem in suitable Sobolev space, the existence of this inverse problem are obtained. Moreover by means of Gronwall inequality, we prove the uniqueness of this inverse problem for small measurement time. For this nonlinear inverse problem, our theoretical results guarantee the solvability for the proposed physical model.

1 Introduction

The time-dependent drift-diffusion model for semiconductor devices under consideration consists of two nonlinear parabolic equations for the electron and

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hole densities n, p, supplemented with an Possion equation for the electrostatic potential ψ, as follows

$$-\nabla\cdot(\nabla\psi) = p - n + f(x), \tag{1}$$
$$n_t - \nabla\cdot J_n = G(n,p), \quad J_n = \nabla n - n\nabla\psi, \tag{2}$$
$$p_t + \nabla\cdot J_p = G(n,p), \quad -J_p = \nabla p + p\nabla\psi, \tag{3}$$

in $\Omega \subseteq R^N (N = 1, 2, 3)$ with the initial condition

$$n(x,0) = n_0, \; p(x,0) = p_0, \qquad x \in \Omega, \tag{4}$$

and the boundary condition

$$\psi(x,t) = \bar{\psi}(x), \; n(x,t) = \bar{n}(x), \; p(x,t) = \bar{p}(x), \; (x,t) \in S_T := \partial\Omega \times (0,T). \tag{5}$$

Here J_n represents the electron current, and J_p is the analogously defined physical quantity of the positively charged holes. $f(x)$ prescribes doping profile characterizing the device under consideration, i.e. the density difference of ionized donors and acceptors. The term $G(n,p) = r(n,p)(1 - np)$ denotes the net recombination-generation rate. Ω is occupied by the semiconductor crystal. The functions $\bar{\psi}, \bar{n}, \bar{p}$ represent the prescribed boundary values of the electrostatic potential and the densities at the Ohmic contact. In many cases of operating devices, these functions may be assumed to be independent of time t.

Our aim in this paper is to reconstruct the doping profile from the following final overdetermination data:

$$\psi(x,T) = \varphi(x), \qquad x \in \Omega. \tag{6}$$

The mix-boundary conditions are considered more reasonable for applications. But the fact that the mixed-boundary conditions prevent us from obtaining high regularity for the solutions gives the difficulty to construct a compact map from doping profile to our measurements. We will pursue this issue in another study. In addition, the regularity of $\partial\Omega$ assumed below is more smooth than the usual one in the physical case. This is because that we need the solution of elliptic equation (1.1) belongs to $L^\infty(0,T;W^{2,p}(\Omega))$ to ensure that $\nabla\psi \in L^\infty(Q_T)$.

In solid-state physics, drift-diffusion model (1.1)-(1.5) are today the most widely used model to describe semiconductor devices. It can be derived from Boltzmann's equation once assumed that the semiconductor devices is in the low injection regime, i.e. for small absolute values of the applied voltage. The direct problem related to this model has been investigated in many papers (see

[1],[12]-[14],[16],[17]). Existence and uniqueness of weak solutions and other properties of solutions were shown.

But identification problems for semiconductor devices, although of increasing technological importance, seem to be poorly understood so far. An important inverse problem in semiconductor devices is the so-called inverse doping profile problem. Because in the application the process modeling [15] only gives a rough estimate of the doping profile, reconstruction of the real doping profile from indirect data becomes an efficient alternative. Recently, such problem gives rise to more and more authors' interest. See for example [4]-[11] and therein. Fang and Ito [10],[18] studied the reconstruction of doping profile, the device parameters from its LBIC (laser-beam-induced current) image. Recently, Burge et al. [4], Burge et al. [6] and Burge et al. [7] investigated the problem of identifying doping profile from indirect measurements of the current or the voltage on a contact. And in [2],[3] the identification method of the discontinuous doping profiles by the stationary voltage-current map was given by Leitão. Several numerical method were applied to this kind of inverse problem in these papers. But these method above are mainly based on a simplified version of the stationary semiconductor model. As far as we know, few works are concerned with the reconstruction of doping profile in a standard time-dependent drift-diffusion model.

For any integer m, q, denote by $W^{m,q}(\Omega)$ the usual Sobolev spaces defined for spatial variable. We need also the following spaces:

$$W_q^{2,1}(Q_T) := \{u; D_x^\alpha D_t^\beta u \in L^q(Q_T), \text{ for all } 0 \le \alpha + 2\beta \le 2\}.$$

Now we can state our inverse problem as follows:

Inverse Problem. For given q such that

$$\begin{cases} \frac{N+2}{2} < q < +\infty, & N = 1, \\ N < q < +\infty, & N \ge 2, \end{cases}$$

determine $(\psi, n, p) \in L^\infty(0, T; W^{2,q}(\Omega)) \times W_q^{2,1}(Q_T) \times W_q^{2,1}(Q_T)$ and $f(x) \in L^q(\Omega)$ from (1)-(5) and the additional measurement (6).

We make the following assumptions throughout this paper.

(H1) The boundary $\partial\Omega \in C^2$;

(H2) $0 \le r(n,p) \le \bar{r} < \infty$ and $r(n,p)$ is a locally Lipschitz continuous function defined for (n,p), that is, there exists a positive constant L such that

$$|r(n_2,p_2) - r(n_1,p_1)| \le L(|n_2 - n_1| + |p_2 - p_1|);$$

(H3) $\bar{n}, \bar{p}, \bar{\psi} \in W^{2,q}(\Omega)$, and $\bar{n}$, $\bar{p} \ge 0$ in Q_T;

(H4) $n_0, p_0 \in W^{2,q}(\Omega)$ and $n_0, p_0 \ge 0$ in Ω;

(H5) $\varphi \in W^{2,q}(\Omega)$ and the following compatibility condition holds:

$$\varphi(x) = \bar{\psi}(x), \quad x \in \partial\Omega. \tag{7}$$

The purpose of the present paper is to establish the existence and uniqueness of our inverse problem for small time T. The main results for our inverse problem in this paper are as follows:

Theorem 1.1. *Let hypotheses (H1)-(H5) hold. Then there exists $T^* > 0$ such that our inverse problem (1)-(6) has a solution $(\psi, n, p, f) \in L^\infty(0, T; W^{2,q}(\Omega)) \times W_q^{2,1}(Q_T) \times W_q^{2,1}(Q_T) \times L^q(\Omega)$ for every $T \in (0, T^*]$.*

Theorem 1.2. *Let hypotheses (H1)-(H5) hold. Then there exists $T^* > 0$ such that the solution $(\psi, n, p, f) \in L^\infty(0, T; W^{2,q}(\Omega)) \times W_q^{2,1}(Q_T) \times W_q^{2,1}(Q_T) \times L^q(\Omega)$ of our inverse problem (1)-(6) is unique for every $T \in (0, T^*]$.*

2 Some results for the direct problem

In this section, we will give the existence and uniqueness of the strongly solution for the direct problem (1.1)-(1.5) which is the basis for constructing the map from doping profile to measurement data. There are many results for the existence and uniqueness problem for the direct problem. See, for example, [12] and [17]. Here the difference with our result and the previous one is that we discuss the existence and uniqueness of solution in the Sobolev space $W_q^{2,1}(Q_T)$ and the assumption that $f(x) \in L^q(\Omega)$ is weaker than the one in [17].

The next existence and uniqueness theorem is our result in this section.

Theorem 2.1. *Let hypotheses (H1-(H5) hold and $f \in L^q(\Omega)$. Then there exists one unique solution $(\psi, n, p) \in L^\infty(0, T; W^{2,q}(\Omega)) \times W_q^{2,1}(Q_T) \times W_q^{2,1}(Q_T)$ of the direct problem constituted by (1)-(5) such that*

$$\|n\|_{L^\infty(Q_T)} + \|p\|_{L^\infty(Q_T)} \le M. \tag{8}$$

Here M is dependent on Ω, T, N, q the known initial and boundary data and $\|f\|_{L^q(\Omega)}$.

To prove Theorem 2.1, we first introduce the following auxiliary problem with the initial and boundary conditions (4) and (5):

$$-\nabla \cdot (\nabla\psi) = p_k - n_k + f(x), \tag{9}$$

$$n_t - \Delta n + \nabla\psi \cdot \nabla n + (-p_k + n_k)n - n_k f(x) = r(n_k, p_k)(1 - np_k), \tag{10}$$

$$p_t - \Delta p - \nabla\psi \cdot \nabla p + (p_k - n_k)p + p_k f(x) = r(n_k, p_k)(1 - n_k p), \tag{11}$$

where $s_k = \min\{k, \max\{s, 0\}\}$ for some positive integer k.

Lemma 2.2. *Let hypotheses (H1-(H5) hold and $f \in L^q(\Omega)$. Then there exists one solution $(\psi, n, p) \in L^\infty(0, T; W^{2,q}(\Omega)) \times W_q^{2,1}(Q_T) \times W_q^{2,1}(Q_T)$ of problem (9)-(11), under initial and boundary conditions (4) and (5).*

Proof. The proof is based on the Leray-Schauder's fixed point theorem [19]. To do this, we set $U = L^q(Q_T) \times L^q(Q_T)$, which is endowed with the following norm

$$\|(n,p)\|_U = \|n\|_{L^q(Q_T)} + \|p\|_{L^q(Q_T)}.$$

Given $(\tilde{n}, \tilde{p}) \in U$ and $0 \le \sigma \le 1$, we consider the following problems:

$$-\nabla \cdot (\nabla \psi) = \tilde{p}_k - \tilde{n}_k + f(x), \tag{12}$$

$$\psi(x,t) = \bar{\psi}(x), \quad (x,t) \in S_T, \tag{13}$$

$$\begin{aligned} &n_t - \Delta n + \sigma \nabla \psi \cdot \nabla n + \sigma(-\tilde{p}_k + \tilde{n}_k + r(\tilde{n}_k, \tilde{p}_k)\tilde{p}_k)n \\ &\quad = \sigma \tilde{n}_k f(x) + \sigma r(\tilde{n}_k, \tilde{p}_k), \end{aligned} \tag{14}$$

$$n(x,t) = \sigma \bar{n}(x), \quad (x,t) \in S_T, \quad n(x,0) = \sigma n_0(x), \quad x \in \Omega \tag{15}$$

and

$$\begin{aligned} &p_t - \Delta p - \sigma \nabla \psi \cdot \nabla p + \sigma(\tilde{p}_k - \tilde{n}_k + r(\tilde{n}_k, \tilde{p}_k)\tilde{n}_k)p \\ &\quad = -\sigma \tilde{p}_k f(x) + \sigma r(\tilde{n}_k, \tilde{p}_k), \end{aligned} \tag{16}$$

$$p(x,t) = \sigma \bar{p}(x), \quad (x,t) \in S_T, \quad p(x,0) = \sigma p_0(x), \quad x \in \Omega. \tag{17}$$

We deduce the existence of a unique strong solution ψ of (12) and (13) belonging to $L^\infty(0,T; W^{2,q}(\Omega))$ from the strong solution theory for elliptic equation [20]. Noting that $L^\infty(0,T; W^{2,q}(\Omega)) \hookrightarrow L^\infty(0,T; W^{1,\infty}(\Omega))$ for $q > N$, there is a unique strong solution n of (14) and (15) with the regularity $n \in W_q^{2,1}(Q_T)$ in terms of ψ, where the $W_q^{2,1}$-norm for n is dependent on Ω, T, k and the known data. Similar results hold for problem (16) and (17).

Thus, the map

$$S : U \times [0,1] \to U, \quad ((\tilde{n}, \tilde{p}), \sigma) \mapsto (n, p) \tag{18}$$

is well defined and compact. When $\sigma = 0$, we easily conclude that

$$S((\tilde{n}, \tilde{p}), 0) = 0, \quad \forall (\tilde{n}, \tilde{p}) \in U.$$

To apply the Leray-Schauder fixed point theorem, we still need to prove that there exists a constant C_0 such that $\|(n,p)\|_U \le C_0$ for all $(n,p) \in U$ satisfying

$(n,p) = S((n,p),\sigma)$. That is to say, for some $\sigma \in [0,1]$, (n,p) satisfies

$$\begin{aligned}
&n_t - \Delta n + \sigma\nabla\psi\cdot\nabla n + \sigma(-p_k + n_k + r(n_k,p_k)p_k)n = \sigma n_k f(x) + \sigma r(n_k,p_k),\\
&n(x,t) = \sigma\bar{n}(x), \quad (x,t)\in S_T, \quad n(x,0) = \sigma n_0(x), \quad x\in\Omega,\\
&p_t - \Delta p - \sigma\nabla\psi\cdot\nabla p + \sigma(p_k - n_k + r(n_k,p_k)n_k)p = -\sigma p_k f(x) + \sigma r(n_k,p_k),\\
&p(x,t) = \sigma\bar{p}(x), \quad (x,t)\in S_T, \quad p(x,0) = \sigma p_0(x), \quad x\in\Omega
\end{aligned}$$

in terms of ψ, which is the unique solution of the following problem

$$\begin{aligned}
&-\nabla\cdot(\nabla\psi) = p_k - n_k + f(x),\\
&\psi(x,t) = \bar{\psi}(x), \quad (x,t)\in S_T.
\end{aligned}$$

Indeed, due to

$$\begin{aligned}
\|\psi\|_{L^\infty(0,T;W^{2,q}(\Omega))} \le& C(\|n_k + p_k + f(x)\|_{L^\infty(0,T;L^q(\Omega))} + \|\bar{\psi}\|_{W^{2,q}(\Omega)})\\
\le& C(2k|\Omega|^{\frac{1}{q}} + \|f(x)\|_{L^q(\Omega)} + \|\bar{\psi}\|_{W^{2,q}(\Omega)})
\end{aligned}$$

and the L^p theory of parabolic equation, we know that there exists a constant C_0 depending on $\Omega, T, k, \bar{r}$ and the initial and boundary data such that

$$\begin{aligned}
\|n\|_{W_q^{2,1}(Q_T)} \le& C(\|\sigma n_k f(x) + \sigma r(n_k,p_k)\|_{L^q(Q_T)})\\
&+ C(\|\sigma\bar{n}\|_{W_q^{2,1}(Q_T)} + \|\sigma n_0\|_{W^{2,q}(\Omega)})\\
\le& \frac{1}{2}C_0.
\end{aligned}$$

Similarly, $\|p\|_{W_q^{2,1}(Q_T)} \le 1/2C_0$. So that $\|(n,p)\|_U \le C_0$. Then the application of Leray-Schauder fixed point theorem gives the existence of the strong solution of problem (9)-(11), under the conditions (4) and (5). □

The following result is of crucial importance to obtain the upper bounds for n, p, whose proof can be found in [13].

Lemma 2.3, *Suppose $\chi(c)$ is nonnegative, nonincreasing function on $[c_0,+\infty]$, and there are positive constants γ and β such that*

$$\chi(\hat{c}) \le M(c)(\hat{c}-c)^{-\gamma}\chi(c)^{1+\beta} \qquad for\ \ all\ \ \hat{c} > c \ge c_0 > 0$$

where the function $M(c)$ is nondecreasing and satisfies

$$0 \le c^{-\gamma}M(c) \le M_0 < +\infty \qquad on\ [c_0,+\infty).$$

Then

$$\chi(c^*) = 0 \qquad for\ \ c^* = 2c_0[1 + 2^{(1+2\beta)/\beta^2}M_0^{(1+\beta)/\beta\gamma}\chi(c_0)^{(1+\beta)/\gamma}].$$

Lemma 2.4. *Let hypotheses (H1)-(H5) hold and $f \in L^q(\Omega)$, then the solution (ψ, n, p) of problem (9)-(11), under conditions (4) and (5), satisfies the following estimate*

$$0 \le n(x,t), p(x,t) \le M, \tag{19}$$

where M is depending on Ω, T, N, q the known initial and boundary data and $\|f\|_{L^q(\Omega)}$, but independent of k.

Proof. Set $N = ne^{-\alpha t}$ and $P = pe^{-\alpha t}$, where α satisfies

$$\alpha > k + \frac{1}{2}\|\nabla\psi\|^2_{L^\infty(Q_T)}.$$

Then (N, P) satisfies

$$N_t + (\alpha - p_k + n_k + r(n_k,p_k)p_k)N - \Delta N + \nabla\psi\cdot\nabla N$$
$$= e^{-\alpha t}\left[n_k f + r(n_k,p_k)\right], \tag{20}$$
$$P_t + (\alpha + p_k - n_k + r(n_k,p_k)n_k)P - \Delta P - \nabla\psi\cdot\nabla P$$
$$= e^{-\alpha t}\left[-p_k f + r(n_k,p_k)\right], \tag{21}$$

and initial and boundary conditions:

$$(e^{\alpha t}N, e^{\alpha t}P) = (\bar{n},\bar{p}), \quad (x,t)\in S_T, \tag{22}$$
$$(N,P) = (n_0,p_0), \quad x\in\Omega,\ t=0. \tag{23}$$

Multiplying (20) by $N^- = \min\{N, 0\}$ and noting that

$$N^-(x,t) = 0,\ (x,t)\in S_T, \quad N^-(x,0) = 0,\ x\in\Omega,$$

we obtain

$$\frac{1}{2}\int_\Omega N^-(t)^2 + \int_0^T\int_\Omega \left|\nabla N^-\right|^2 + (\alpha - p_k + n_k + r(n_k,p_k)p_k)\int_0^T\int_\Omega N^{-2}$$
$$= -\int_0^T\int_\Omega N^-\nabla\psi\cdot\nabla N + \int_0^T\int_\Omega e^{-\alpha t}\left[n_k f + r(n_k,p_k)\right]N^-$$
$$\le \frac{1}{2}\int_0^T\int_\Omega |\nabla N^-|^2 + \frac{1}{2}\|\nabla\psi\|^2_{L^\infty(Q_T)}\int_0^T\int_\Omega N^{-2} + \int_0^T\int_\Omega r(n_k,p_k)N^-,$$

where we have used that $\int_0^T\int_\Omega n_k f N^- = 0$. By $r(n_k,p_k)\ge 0$ and the choice of α, we obtain

$$\int_\Omega N^-(t)^2 + \int_0^T\int_\Omega \left|\nabla N^-\right|^2 \le 0.$$

That is $n \geq 0$. $p \geq 0$ follows by a similar way.

Now we prove that $n, p \leq M$ with some positive constant M, which is independent of k. Let $s \geq c_0 = \max\{\max_\Omega\{n_0, p_0\}, \max_\Omega\{\bar{n}, \bar{p}\}\}$. Multiplying (10) and (11) by $(n-s)^+$ and $(p-s)^+$ respectively, and then integrating on Q_t we obtain

$$\begin{aligned}
&\frac{1}{2}\int_\Omega \left[(n-s)^+(t)^2 + (p-s)^+(t)^2\right] + \int_0^t \int_\Omega \left[\left|\nabla(n-s)^+\right|^2 + \left|\nabla(p-s)^+\right|^2\right] \\
\leq& -\int_0^t \int_\Omega \nabla\psi \cdot \left[(n-s)^+\nabla n - (p-s)^+\nabla p\right] \\
&+ \int_0^t \int_\Omega (p_k - n_k)\left[n(n-s)^+ - p(p-s)^+\right] \\
&+ \int_0^t \int_\Omega r(n_k, p_k)\left[(n-s)^+ + (p-s)^+\right] \\
&+ \int_0^t \int_\Omega f(x)\left[n_k(n-s)^+ - p_k(p-s)^+\right] \\
\leq& \int_0^t \int_\Omega (p_k - n_k)\left[\frac{(n-s)^{+2}}{2} - \frac{(p-s)^{+2}}{2}\right] + \bar{r}\int_0^t \int_\Omega \left[(n-s)^+ + (p-s)^+\right] \\
&+ \int_0^t \int_\Omega f(x)\left[\left(n_k(n-s)^+ - \frac{(n-s)^{+^2}}{2}\right) - \left(p_k(p-s)^+ - \frac{(p-s)^{+^2}}{2}\right)\right] \\
&+ s\int_0^t \int_\Omega (p_k - n_k)\left[(n-s)^+ - (p-s)^+\right] \\
\leq& \int_0^t \int_\Omega |\bar{r} + sf(x)|\left[(n-s)^+ + (p-s)^+\right] \\
&+ \frac{3}{2}\int_0^t \int_\Omega |f(x)|\left[(n-s)^{+^2} + (p-s)^{+^2}\right] \\
=& I_1 + I_2,
\end{aligned} \tag{24}$$

where we have used that $(p_k - n_k)((n-s)^+ - (p-s)^+) \leq 0$ and

$$0 \leq \theta_k(\theta - s)^+ \leq \theta(\theta - s)^+ = (\theta - s)^{+^2} + s(\theta - s)^+$$

for $\theta = n, p$ in the last inequality.

Hölder inequality gives

$$\begin{aligned}
I_1 \leq& t^{\frac{1}{q}}\left(|\Omega|^{\frac{1}{q}}\bar{r} + s\|f(x)\|_{L^q(\Omega)}\right)\|(n-s)^+ + (p-s)^+\|_{L^{\frac{2(N+2)}{N}}(Q_t)} \\
&\times |Q_t \cap [n > s, p > s]|^{\frac{qN+4q-2(N+2)}{2q(N+2)}},
\end{aligned} \tag{25}$$

where $|\cdot|$ denotes the Lebesgue measure and $[n > s, p > s] = \{(x,t)|n(x,t) > s, p(x,t) > s\}$. Similarly,

$$\begin{aligned} I_2 \leq & \frac{3}{2}\|f(x)\|_{L^q(Q_t)}\|(n-s)^{+^2} + (p-s)^{+^2}\|_{L^{\frac{q}{q-1}}(Q_t)} \\ \leq & \frac{3}{2}t^{\frac{1}{q}}(t|\Omega|)^{\frac{2q-(N+2)}{(N+2)q}}\|f(x)\|_{L^q(\Omega)}\|(n-s)^+ + (p-s)^+\|^2_{L^{\frac{2(N+2)}{N}}}. \end{aligned} \tag{26}$$

Inserting (25) and (26) into (24), together with the following embedding equality

$$\|u\|^2_{L^{\frac{2(N+2)}{2}}(Q_{t_0})} \leq C(\sup_{0\leq t\leq t_0}\|u(\cdot,t)\|^2_{L^2(\Omega)} + \|\nabla u\|^2_{L^2(Q_{t_0})})$$

yields

$$\begin{aligned} \|(n-s)^+ + (p-s)^+\|_{L^{\frac{2(N+2)}{N}}(Q_{t_0})} \leq & Ct_0{}^{\frac{1}{q}}\left(|\Omega|^{\frac{1}{q}}\bar{r} + s\|f(x)\|_{L^q(\Omega)}\right) \\ & \times |Q_{t_0} \cap [n > s, p > s]|^{\frac{qN+4q-2(N+2)}{2q(N+2)}}, \end{aligned} \tag{27}$$

if we choose t_0 sufficiently small. Here C is only depending on Ω, T, but independent of k.

On the other hand, we have

$$\|(n-s)^+ + (p-s)^+\|_{L^{\frac{2(N+2)}{N}}(Q_{t_0})} \geq (\hat{s}-s)|Q_{t_0} \cap [n > \hat{s}, p > \hat{s}]|^{\frac{N}{2(N+2)}} \tag{28}$$

for all $c_0 \leq s < \hat{s}$.

Let $\chi(s) = |Q_{t_0} \cap [n > s, p > s]|$. From (27) and (28), we obtain for $c_0 \leq s < \hat{s}$

$$\chi(\hat{s}) \leq m(s)(\hat{s}-s)^{-\gamma}\chi(s)^{1+\beta},$$

where

$$\begin{aligned} m(s) &= \left[Ct_0{}^{\frac{1}{q}}\left(|\Omega|^{\frac{1}{q}}\bar{r} + s\|f(x)\|_{L^q(\Omega)}\right)\right]^{\frac{2(N+2)}{N}} \\ \beta &= \frac{qN+4q-2(N+2)}{qN} - 1, \quad \gamma = \frac{2(N+2)}{N}. \end{aligned}$$

Noticing that

$$0 \leq s^{-\gamma}m(s) \leq m_0 := \left[Ct_0^{\frac{1}{q}}\left(|\Omega|^{\frac{1}{q}}\bar{r}c_0^{-1} + \|f\|_{L^q(\Omega)}\right)\right]^{\gamma} < +\infty \quad \text{on } [c_0, +\infty),$$

we get the $L^\infty(Q_{t_0})$ estimates for n and p uniformly in k by Lemma 2.3. Repeating the above procedure, we can prove (19) and then complete the proof. □

Proof of Theorem 2.1. Note that M is uniformly in k. So we can take k large enough in (9)-(11) to prove the existence result for problem (1)-(5). Moreover by using the standard method (for details, see [12] or [17]), we can obtain the uniqueness of solution (ψ, n, p) of (1)-(5). □

3 Existence result for the inverse problem

This section is devoted to prove the existence of our inverse problem (1)-(6). We introduce a nonlinear operator equation whose solvability can deduce our desired existence result. In order to obtain the solvability of this nonlinear operator equation, we need a fixed point arguments by means of the Schauder's fixed theorem.

We define the set D as

$$D = \left\{ f \in L^q(\Omega) : \|f\|_{L^q(\Omega)} \leq R \right\},$$

where R is a large constant which will be specified below. In addition, we define a nonlinear operator $\mathscr{A}$ as

$$\mathscr{A} : D \to L^q(\Omega)$$

with the values

$$(\mathscr{A} f)(x) = n(x,T) - p(x,T)$$

with (ψ, n, p) solution of the direct problem (1)-(5) in terms of f. Theorem 2.1 shows that the operator $\mathscr{A}$ is well defined. Furthermore we introduce a nonlinear operator equation of the second kind of f:

$$f = \mathscr{A} f + \zeta, \tag{29}$$

where $\zeta(x) = -\Delta\varphi(x)$.

In the following Lemma we establish an interconnection between the nonlinear operator equation (29) and the inverse problem (1)-(6).

Lemma 3.1. *Let hypotheses (H1)-(H5) hold. If equation (29) has a solution lying within D, then there exists a solution of the inverse problem (1)-(6).*

Proof. By assumption, the nonlinear equation (29) has a solution lying within D. We denote this solution by $f(x)$ and substitute it into the direct problem (1)-(5). Theorem 2.1 ensure that the direct problem constituted by (1)-(5) has a unique solution $(\psi, n, p) \in L^\infty(0,T; W^{2,q}(\Omega)) \times W_q^{2,1}(Q_T) \times W_q^{2,1}(Q_T)$.

Finally, let us show that the function ψ satisfies the overdetermination condition (6). From the compatibility condition (7) and the definition of $\mathscr{A}$, we know that $\psi^* = \psi(x,T) - \varphi(x)$ satisfies

$$-\Delta\psi^* = 0, \quad x \in \Omega,$$
$$\psi^* = 0, \quad x \in \partial\Omega.$$

Thus, it is evident that the function ψ satisfies the final overdetermination (6), thereby justifying the assertion of this lemma. □

In order to establish the solvability of equation (29), we need a careful analysis of the nonlinear operator $\mathscr{A}$. The following Lemma gives the complete continuity of operator $\mathscr{A}$.

Lemma 3.2 *Let hypotheses (H1)-(H5) hold. Then the operator $\mathscr{A}$ is completely continuous on D.*

Proof. Here without loss of generality, we assume $N \geq 3$ and the $N \leq 2$ case is easier. Due to (8) and the standard parabolic equations theory, we know that there exists a constants C_1 depending on Ω, T, M and the known data such that

$$\|n\|_{W_q^{2,1}(Q_T)} + \|p\|_{W_q^{2,1}(Q_T)} + \|\psi\|_{L^\infty(0,T;W^{2,q}(\Omega))} \leq C_1,$$

if $f \in D$. This implies that $\|\nabla\psi\|_{L^\infty(Q_T)} \leq C_1$, since $q > N$.

In order to obtain the compactness of $\mathscr{A}$, we need to prove $n(x,T), p(x,T) \in W^{1,q}(\Omega)$. To do this, we multiply (2) by $|\nabla n|^{q-2}n_t$ and integrate it over Ω,

$$\begin{aligned}
&\int_\Omega |\nabla n|^{q-2}n_t^2 + \frac{1}{q}\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega |\nabla n|^q \leq \\
&-\int_\Omega \nabla n \cdot \nabla\left(|\nabla n|^{q-2}\right)n_t \quad -\int_\Omega \nabla n \cdot \nabla\psi|\nabla n|^{q-2}n_t \\
&-\int_\Omega n\Delta\psi|\nabla n|^{q-2}n_t + \int_\Omega r(n,p)(1-np)|\nabla n|^{q-2}n_t \\
&\quad = I_1 + \cdots + I_4.
\end{aligned} \tag{30}$$

Here we note that $n_t = (\bar{n})_t = 0$ on $\partial\Omega$.

Next we will estimate $I_1, \cdots, I_4$ terms by terms. It is easy to verify that

$$\begin{aligned}
\nabla n \cdot \left(\nabla|\nabla n|^{q-2}\right) =&(q-2)\sum_{i=1}^N \left(n_{x_i} \cdot \sum_{j=1}^N |n_{x_j}|^{q-4}n_{x_j}n_{x_ix_j}\right) \\
\leq&(q-2)|\nabla n|^{q-2}\sum_{i,j=1}^N |n_{x_ix_j}|.
\end{aligned}$$

Then, by Hölder inequality we obtain

$$
\begin{aligned}
I_1 \leq & (q-2)\int_\Omega \left(|\nabla n|^{q-2} n_t \sum_{i,j=1}^{N} |n_{x_i x_j}| \right) \\
\leq & \frac{1}{2}\int_\Omega |\nabla n|^{q-2} n_t^2 + \frac{(q-2)^3}{2q}\int_\Omega |\nabla n|^q + \frac{(q-2)^2}{q}\|n\|^q_{W^{2,q}(\Omega)}. \qquad (31)
\end{aligned}
$$

By the Young inequality,

$$
\begin{aligned}
I_2 + I_3 + I_4 \leq & \frac{1}{2}\int_\Omega |\nabla n|^{q-2} n_t^2 + \left(\frac{3}{2}\|\nabla\psi\|^2_{L^\infty(\Omega)} + \frac{3(q-2)M^2}{2q}\right)\int_\Omega |\nabla n|^q \\
& + \frac{3M^2}{q}\|\Delta\psi\|^q_{L^q(\Omega)} + \frac{3\bar{r}^2(1+M^2)^2}{2}\int_\Omega |\nabla n|^{q-2}. \qquad (32)
\end{aligned}
$$

Substituting (31)-(32) into (30), we have

$$
\begin{aligned}
\frac{1}{q}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla n\|^q_{L^q(\Omega)} \leq & C_2\|\nabla n\|^q_{L^q(\Omega)} + C_3\|\nabla n\|^{q-2}_{L^q(\Omega)} \\
& + \frac{(q-2)^2}{q}\|n\|^q_{W^{2,q}(\Omega)} + \frac{3M^2}{q}\|\Delta\psi\|^q_{L^q(\Omega)}, \qquad (33)
\end{aligned}
$$

where

$$
\begin{aligned}
C_2 &= \frac{(q-2)^3}{2q} + \frac{3C_1^2}{2} + \frac{3(q-2)M^2}{2q}, \\
C_3 &= \frac{3\bar{r}^2(1+M^2)^2}{2}|\Omega|^{\frac{2}{q}}.
\end{aligned}
$$

From (33), we obtain

$$
\|\nabla n(t)\|^q_{L^q(\Omega)} \leq (qC_2 + (q-2)C_3)\int_0^t \|\nabla n(\tau)\|^q_{L^q(\Omega)}\mathrm{d}\tau + C_4. \qquad (34)
$$

Here

$$
C_4 = \left(3(q-2)^2 + 3M^2\right) C_1^q + 2C_3 T + \|n_0\|^q_{L^q(\Omega)}.
$$

Hence, applying the Gronwall inequality to (34), we obtain

$$
\max_{0\geq t\leq T}\|\nabla n(t)\|_{L^q(\Omega)} \leq [C_4\exp\{(qC_2 + (q-2)C_3)T\}]^{\frac{1}{q}} := C_5, \qquad (35)
$$

where C_5 is depending on $\Omega, T, q, \bar{r}, M$ and the known initial and boundary data. Thus we obtain $n(x,T) \in W^{1,q}(\Omega)$. A similar result holds for p. Since the injection $W^{1,q}(\Omega) \hookrightarrow L^q(\Omega)$ is compact, we conclude that $\mathscr{A}$ is compact on D.

Next we will prove that $\mathscr{A}$ is continuous on D. Let $f_\nu, f \in D$ such that $f_\nu \to f$ in $L^q(\Omega)$. Define $(\mathscr{A} f_\nu)(x) = n_\nu(x,T) - p_\nu(x,T)$, where (ψ_ν, n_ν, p_ν) is the unique solution of the direct problem (1)-(5) corresponding to f_ν. And let (ψ, n, p) be the solution of the same problem corresponding to f. Set

$$h(n_\nu, p_\nu, n, p) = r(n_\nu, p_\nu)(1 - n_\nu p_\nu) - r(n,p)(1-np).$$

By the Lipschitz continuity of function r, we have

$$|h(n_\nu, p_\nu, n, p)| \leq (L + \bar{r}M + M^2 L)(|n_\nu - n| + |p_\nu - p|).$$

It is obvious that the functions $n_\nu - n, p_\nu - p, \psi_\nu - \psi$ satisfy

$$\begin{aligned}
&-\Delta(\psi_\nu - \psi) = (p_\nu - p) + (n_\nu - n) + (f_\nu - f),\\
&(n_\nu - n)_t - \Delta(n_\nu - n) + \nabla(n_\nu - n)\cdot\nabla\psi_\nu + \nabla n \cdot \nabla(\psi_\nu - \psi) = h(n_\nu, p_\nu, n, p),\\
&(p_\nu - p)_t - \Delta(p_\nu - p) - \nabla(p_\nu - p)\cdot\nabla\psi_\nu - \nabla p \cdot \nabla(\psi_\nu - \psi) = h(n_\nu, p_\nu, n, p),
\end{aligned}$$

and the initial and boundary conditions:

$$\begin{aligned}
&(n_\nu - n)(x,0) = (p_\nu - p)(x,0) = 0, \quad x \in \Omega,\\
&(\psi_\nu - \psi)(x,t) = (n_\nu - n)(x,t) = (p_\nu - p)(x,t) = 0, \quad (x,t) \in S_T.
\end{aligned}$$

We multiply the equation of $n_\nu - n$ by $|n_\nu - n|^{q-2}(n_\nu - n)$, and integrate with respect to x on Ω. Then we have

$$\begin{aligned}
&\frac{1}{q}\frac{\mathrm{d}}{\mathrm{d}t}\|n_\nu - n\|^q_{L^q(\Omega)} + \frac{4(q-1)}{q^2}\int_\Omega |\nabla|n_\nu - n|^{\frac{q}{2}}|^2\\
=&-\frac{2}{q}\int_\Omega |n_\nu - n|^{\frac{q}{2}}\nabla\psi_\nu \cdot \nabla|n_\nu - n|^{\frac{q}{2}} + \int_\Omega |n_\nu - n|^{q-2}(n_\nu - n)\\
&\times[-\nabla n\cdot\nabla(\psi_\nu - \psi) + h]\\
\leq&\frac{2(q-1)}{q^2}\int_\Omega |\nabla|n_\nu - n|^{\frac{q}{2}}|^2 + \||n_\nu - n|^{q-1}\|_{L^{\frac{q}{q-1}}(\Omega)}\|\nabla n\|_{L^q(\Omega)}\\
&\times\|\nabla(\psi_\nu - \psi)\|_{L^\infty(\Omega)} + \left[\frac{\|\nabla\psi_\nu\|^2_{L^\infty(\Omega)}}{2(q-1)} + 2(L + \bar{r}M + M^2 L)\right]\\
&\times\left(\|n_\nu - n\|^q_{L^q(\Omega)} + \|p_\nu - p\|^q_{L^q(\Omega)}\right).
\end{aligned} \tag{36}$$

A standard elliptic estimate gives

$$\begin{aligned}
\|\nabla(\psi_\nu - \psi)\|_{L^\infty(\Omega)} \leq& \|\psi_\nu - \psi\|_{W^{2,q}(\Omega)}\\
\leq& \|n_\nu - n\|_{L^q(\Omega)} + \|p_\nu - p\|_{L^q(\Omega)} + \|f_\nu - f\|_{L^q(\Omega)},
\end{aligned} \tag{37}$$

since $q > N$. Substituting (37) into (36) and integrating with respect to t, we have

$$\begin{aligned} \|(n_\nu - n)(t)\|^q_{L^q(\Omega)} \le{}& C_1^q \|f_\nu - f\|^q_{L^q(\Omega)} \\ &+ C_6 \int_0^t \left(\|(n_\nu - n)(\tau)\|^q_{L^q(\Omega)} + \|(p_\nu - p)(\tau)\|^q_{L^q(\Omega)} \right) \mathrm{d}\tau, \end{aligned} \tag{38}$$

where

$$C_6 = \frac{qC_1^2}{2(q-1)} + 2q\left(L + \bar{r}M + M^2 L\right) + (q-1) + C_1^q.$$

A estimate similar to (38) holds for $p_\nu - p$. So

$$\begin{aligned} \|(n_\nu - n)(t)\|^q_{L^q(\Omega)} + \|(p_\nu - p)(t)\|^q_{L^q(\Omega)} \le{}& 2C_1^q \|f_\nu - f\|^q_{L^q(\Omega)} \\ &+ 2C_6 \int_0^t \left(\|(n_\nu - n)(\tau)\|^q_{L^q(\Omega)} + \|(p_\nu - p)(\tau)\|^q_{L^q(\Omega)} \right) \mathrm{d}\tau. \end{aligned} \tag{39}$$

By the Gronwall inequality,

$$\begin{aligned} \|(n_\nu - n)(T)\|^q_{L^q(\Omega)} + \|(p_\nu - p)(T)\|^q_{L^q(\Omega)} \le{}& 2C_1^q \|f_\nu - f\|^q_{L^q(\Omega)} \exp\left(2C_6 T\right) \\ &\to 0. \end{aligned}$$

That is $\mathscr{A} f_\nu \to \mathscr{A} f$ in $L^q(\Omega)$, which implies that $\mathscr{A}$ is continuous on D.

From above argument, we conclude that $\mathscr{A}$ is completely continuous on D. □

Now we give the proof of Theorem 1.1. Throughout this proof, we will use C to denote a positive constant independent of R, which may be different from line to line.

Proof of Theorem 1.1. For given $f \in D$, let

$$\mathscr{B} f = \mathscr{A} f + \zeta.$$

Because $\mathscr{A}$ is completely continuous on D by Lemma 3.2, it seems clear that the operator $\mathscr{B}$ is completely continuous as the composition of a nonlinear completely continuous operator and a linear bounded one. In order to apply the Schauder's fixed theorem, we need to derive an priori estimate for the nonlinear operator $\mathscr{B}$ on D.

Multiplying (2) and (3) by $n^{q-1} - \bar{n}^{q-1}$ and $p^{q-1} - \bar{p}^{q-1}$, respectively, and adding them together, we get

$$\begin{aligned}
&\frac{1}{q}\int_\Omega (n^q+p^q)+\frac{4(q-1)}{q^2}\int_0^t\int_\Omega \left[|\nabla(n^{\frac{q}{2}})|^2+|\nabla(p^{\frac{q}{2}})|^2\right]\\
=&\frac{1}{q}\int_\Omega (n_0^q+p_0^q)+\int_\Omega \left[\bar{n}^{q-1}(n-n_0)+\bar{p}^{q-1}(p-p_0)\right]\\
&+\int_0^t\int_\Omega \left[\nabla\psi\cdot\left(n\nabla(n^{q-1}-\bar{n}^{q-1})-p\nabla(p^{q-1}-\bar{p}^{q-1})\right)\right]\\
&+\int_0^t\int_\Omega \left[\nabla n\cdot\nabla\left(\bar{n}^{q-1}\right)+\nabla p\cdot\nabla\left(\bar{p}^{q-1}\right)\right]\\
&+\int_0^t\int_\Omega R(n,p)\left[\left(n^{q-1}-\bar{n}^{q-1}\right)+\left(p^{q-1}-\bar{p}^{q-1}\right)\right]\\
=&J_1+\cdots+J_5. \qquad (40)
\end{aligned}$$

By the equation of ψ and Hölder inequality, we obtain

$$\begin{aligned}
J_3=&\frac{q-1}{q}\int_0^t\int_\Omega \nabla\psi\cdot\left[\nabla\left(n^q-\bar{n}^q\right)-\nabla\left(p^q-\bar{p}^q\right)\right]\\
&+\int_0^t\int_\Omega \nabla\psi\cdot\left[(\bar{n}-n)\nabla\left(\bar{n}^{q-1}\right)-(\bar{p}-p)\nabla\left(\bar{p}^{q-1}\right)\right]\\
\le&\frac{q-1}{q}\int_0^t\int_\Omega (p-n+f)\left[(n^q-p^q)-(\bar{n}^q-\bar{p}^q)\right]\\
&+C\int_0^t \|\nabla\psi\|_{L^2(\Omega)}\left[\|n-\bar{n}\|_{L^2(\Omega)}+\|p-\bar{p}\|_{L^2(\Omega)}\right]\\
\le&C\left[1+t\|f\|_{L^q(\Omega)}+\int_0^t\left(\|n\|^q_{L^q(\Omega)}+\|p|^q_{L^q(\Omega)}\right)\right]\\
&+\frac{q-1}{q}\|f\|_{L^q(\Omega)}\int_0^t\left(\|n\|^q_{L^{\frac{q^2}{q-1}}(\Omega)}+\|p\|^q_{L^{\frac{q^2}{q-1}}(\Omega)}\right)\\
&+C\|f\|_{L^2(\Omega)}\int_0^t\left(\|n\|_{L^2(\Omega)}+\|p\|_{L^2(\Omega)}\right), \qquad (41)
\end{aligned}$$

where we have used that $(p-n)(n^q-p^q)\le 0$ and

$$\|\nabla\psi\|_{L^2(\Omega)}\le\|p\|_{L^2(\Omega)}+\|n\|_{L^2(\Omega)}+\|f\|_{L^2(\Omega)}+\|\bar{\psi}\|_{H^1(\Omega)}.$$

Due to the Gagliardo-Nirenberg inequality [21], we get

$$\|n\|^q_{L^{\frac{q^2}{q-1}}(\Omega)}=\|n^{\frac{q}{2}}\|^2_{L^{\frac{2q}{q-1}}(\Omega)}\le\|n^{\frac{q}{2}}\|^{2\lambda}_{L^2(\Omega)}\|\nabla(n^{\frac{q}{2}})\|^{2(1-\lambda)}_{L^2(\Omega)} \qquad (42)$$

where $0 < 1-\lambda = N(\frac{1}{2} - \frac{q-1}{2q}) < 1$. Substituting the estimate (42) for n and a similar estimate for p into (41), and applying the Hölder inequality, we get

$$J_3 \leq C\left[1 + t\|f\|_{L^q(\Omega)} + t\|f\|_{L^q(\Omega)}^{\frac{q}{q-1}} + \|f\|_{L^q(\Omega)}^{\frac{1}{\lambda}} \int_0^t \left(\|n\|_{L^q(\Omega)}^q + \|p\|_{L^q(\Omega)}^q\right)\right] + \frac{2(q-1)}{q^2} \int_0^t \left(\|\nabla(n^{\frac{q}{2}})\|_{L^2(\Omega)}^2 + \|\nabla(p^{\frac{q}{2}})\|_{L^2(\Omega)}^2\right). \tag{43}$$

In addition, from the nonnegativity of n and p and Hölder inequality we have

$$J_4 \leq \frac{1}{4} \int_0^t \left(\|\nabla n\|_{L^2(\Omega)}^2 + \|\nabla p\|_{L^2(\Omega)}^2\right) + C \tag{44}$$

and

$$J_5 \leq \int_0^t \left(\|n\|_{L^q(\Omega)}^q + \|p\|_{L^q(\Omega)}^q\right) + C. \tag{45}$$

From (40), (43)-(45), we get

$$\frac{1}{q}\int_\Omega (n^q + p^q) + \frac{2(q-1)}{q^2} \int_0^t \int_\Omega \left[|\nabla(n^{\frac{q}{2}})|^2 + |\nabla(p^{\frac{q}{2}})|^2\right] \leq C\left[1 + t\|f\|_{L^q(\Omega)} + t\|f\|_{L^q(\Omega)}^{\frac{q}{q-1}} + \|f\|_{L^q(\Omega)}^{\frac{1}{\lambda}} \int_0^t \left(\|n\|_{L^q(\Omega)}^q + \|p\|_{L^q(\Omega)}^q\right)\right] + \frac{1}{4}\int_0^t \left(\|\nabla n\|_{L^2(\Omega)}^2 + \|\nabla p\|_{L^2(\Omega)}^2\right). \tag{46}$$

Especially, when $q = 2$, estimate (46) turns to

$$\frac{1}{2}\int_\Omega (n^2 + p^2) + \frac{1}{2}\int_0^t \int_\Omega \left[|\nabla n|^2 + |\nabla p|^2\right] \leq C\left[1 + t\|f\|_{L^q(\Omega)} + t\|f\|_{L^q(\Omega)}^2 + \|f\|_{L^q(\Omega)}^{\frac{4-N}{4}} \int_0^t \left(\|n\|_{L^2(\Omega)}^2 + \|p\|_{L^2(\Omega)}^2\right)\right] + \frac{1}{4}\int_0^t \left(\|\nabla n\|_{L^2(\Omega)}^2 + \|\nabla p\|_{L^2(\Omega)}^2\right). \tag{47}$$

Then adding (46) and (47) leads to

$$\|n(t)\|_{L^q(\Omega)}^q + \|p(t)\|_{L^q(\Omega)}^q \leq C\left(1 + t\|f\|_{L^q(\Omega)} + t\|f\|_{L^q(\Omega)}^{\frac{q}{q-1}} + t\|f\|_{L^q(\Omega)}^2\right) + C\left(\|f\|_{L^q(\Omega)}^{\frac{1}{\lambda}} + \|f\|_{L^2(\Omega)}^{\frac{4-N}{4}}\right)\int_0^t \left(\|n\|_{L^q(\Omega)}^q + \|p\|_{L^q(\Omega)}^q\right). \tag{48}$$

Thus, the application of Gronwall inequality gives that

$$\sup_{0\leq t\leq T}\left(\|n(t)\|_{L^q(\Omega)}+\|p(t)\|_{L^q(\Omega)}\right)$$
$$\leq\left\{C(1+TR+TR^2+TR^{\frac{q}{q-1}})\exp\left[C\left(R^{\frac{1}{\lambda}}+R^{\frac{4-N}{4}}\right)T\right]\right\}^{\frac{1}{q}}. \qquad (49)$$

From (49) it follows that for arbitrary $f\in D$, there exists a sufficiently small $T^*>0$ such that

$$\|\mathscr{B}f\|_{L^q(\Omega)}\leq\|n(x,T)\|_{L^q(\Omega)}+\|q(x,T)\|_{L^q(\Omega)}+\|\zeta\|_{L^q(\Omega)}\leq R,$$

if we choose R such that $R>2\max\{C^{\frac{1}{q}},\|\zeta\|_{L^q(\Omega)}\}$ and $T\leq T^*$.

This shows that the nonlinear operator $\mathscr{B}$ is completely continuous and carries the closed bounded set D into itself. Schauder's fixed pointed theorem implies that $\mathscr{B}$ has a fixed pointed lying within D. Then Lemma 3.1 yields that there exists a solution of the inverse problem (1)-(6), thereby completing the proof of the theorem. □

4 Uniqueness result for the inverse problem

Now we prove the uniqueness of the solution of our inverse problem (1)-(6) for small time T.

Proof of Theorem 1.2. Suppose that (ψ_1,n_1,p_1,f_1) and (ψ_2,n_2,p_2,f_2) are two solutions of our inverse problem (1)-(6) and define $(\hat\psi,\hat n,\hat p,\hat f)=(\psi_1-\psi_2,n_1-n_2,p_1-p_2,f_1-f_2)$, $G_i=G(n_i,p_i),i=1,2$ and $\hat G=G_1-G_2$. Then $(\hat\psi,\hat n,\hat p,\hat f)$ satisfies

$$\hat n_t-\Delta\hat n+\nabla\cdot(n_1\nabla\hat\psi+\hat n\nabla\psi_2)=\hat G, \qquad (50)$$
$$\hat p_t-\Delta\hat p-\nabla\cdot(p_1\nabla\hat\psi+\hat p\nabla\psi_2)=\hat G, \qquad (51)$$
$$-\Delta\hat\psi=\hat p-\hat n+\hat f, \qquad (52)$$
$$\hat f=\hat n(x,T)-\hat p(x,T) \qquad (53)$$

in Q_T, with initial and boundary conditions

$$\hat\psi=\hat n=\hat p=0,\quad \text{on }\partial\Omega,\quad \hat n(\cdot,0)=\hat p(\cdot,0)=0,\quad \text{in }\Omega. \qquad (54)$$

Multiplying (50) and (51) by $\hat n$ and $\hat p$ respectively, and adding these two equalities, we have

$$\begin{aligned}&\frac{1}{2}\int_\Omega \left(|\hat{n}(t)|^2+|\hat{p}(t)|^2\right)+\int_0^t\int_\Omega \left(|\nabla\hat{n}|^2+|\nabla\hat{p}|^2\right)\\ \le&\int_0^t\int_\Omega \nabla\hat{\psi}\cdot(n_1\nabla\hat{n}-p_1\nabla\hat{p})+\int_0^t\int_\Omega \nabla\psi_2\cdot(\hat{n}\nabla\hat{n}-\hat{p}\nabla\hat{p})+\int_0^t\int_\Omega \hat{G}(\hat{n}+\hat{p})\\ &=K_1+K_2+K_3. \qquad (55)\end{aligned}$$

By Young's inequality,

$$\begin{aligned}K_1\le&(\|n_1\|^2_{L^\infty(Q_T)}+\|p_1\|^2_{L^\infty(Q_T)})\|\nabla\hat{\psi}\|^2_{L^2(Q_T)}\\ &+\frac{1}{4}(\|\nabla\hat{n}\|^2_{L^2(Q_t)}+\|\nabla\hat{p}\|^2_{L^2(Q_t)}), \qquad (56)\end{aligned}$$

and

$$\begin{aligned}K_2\le&\|\nabla\psi_2\|^2_{L^\infty(Q_T)}(\|\hat{n}\|^2_{L^2(Q_t)}+\|\hat{p}\|^2_{L^2(Q_t)})\\ &+\frac{1}{4}(\|\nabla n\|^2_{L^2(Q_t)}+\|\nabla p\|^2_{L^2(Q_t)}). \qquad (57)\end{aligned}$$

The Lipschitz continuity of r in (n,p) implies

$$|\hat{G}|\le\bar{r}\|p_1\|_{L^\infty(Q_T)}\hat{n}+\bar{r}\|n_2\|_{L^\infty(Q_T)}\hat{p}+L\|n_2\|_{L^\infty(Q_T)}\|p_2\|_{L^\infty(Q_T)}(\hat{n}+\hat{p}).$$

Hence,

$$\begin{aligned}K_3\le&\left[\frac{2}{3}\bar{r}\left(\|p_1\|_{L^\infty(Q_T)}+\|n_2\|_{L^\infty(Q_T)}\right)+2L\|n_2\|_{L^\infty(Q_T)}\|p_2\|_{L^\infty(Q_T)}\right]\\ &\times(\|\hat{n}(t)\|^2_{L^2(Q_t)}+\|\hat{p}(t)\|^2_{L^2(Q_t)}). \qquad (58)\end{aligned}$$

In addition, from the elliptic equation (52) and (53), we get

$$\begin{aligned}\|\nabla\hat{\psi}(t)\|^2_{L^2(Q_t)}\le&\|\hat{p}(t)\|^2_{L^2(Q_t)}+\|\hat{n}(t)\|^2_{L^2(Q_t)}\\ &+t\left(\|\hat{n}(x,T)\|^2_{L^2(\Omega)}+\|\hat{p}(x,T)\|^2_{L^2(\Omega)}\right) \qquad (59)\end{aligned}$$

Gathering (55)-(59), we obtain

$$\begin{aligned}\|\hat{n}(t)\|^2_{L^2(\Omega)}+\|\hat{p}(t)\|^2_{L^2(\Omega)}\le&C_1\left(\|\hat{n}(x,T)\|^2_{L^2(\Omega)}+\|\hat{p}(x,T)\|^2_{L^2(\Omega)}\right)t\\ &+C_2\int_0^t\left(\|\hat{n}(\tau)\|^2_{L^2(\Omega)}+\|\hat{p}(\tau)\|^2_{L^2(\Omega)}\right)\mathrm{d}\tau, \qquad (60)\end{aligned}$$

where

$$\begin{aligned} C_1 &:= 2(\|n_1\|^2_{L^\infty(\Omega)} + \|p_1\|^2_{L^\infty(\Omega)}), \\ C_2 &:= C_{16} + 2\Big(\|\nabla\psi_2\|^2_{L^\infty(Q_T)} + \frac{2}{3}\bar{r}\left(\|p_1\|_{L^\infty(Q_T)} + \|n_2\|_{L^\infty(Q_T)}\right) \\ &\quad +2L\|n_2\|_{L^\infty(Q_T)}\|p_2\|_{L^\infty(Q_T)}\Big). \end{aligned}$$

Therefore, the Gronwall inequality yields that for all $t \in [0, T]$

$$\|\hat{n}(t)\|_{L^2(\Omega)} + \|\hat{p}(t)\|_{L^2(\Omega)} \leq C_1\left(\|\hat{n}(x,T)\|_{L^2(\Omega)} + \|\hat{p}(x,T)\|_{L^2(\Omega)}\right) Te^{C_2 t}. \quad (61)$$

From (61), we conclude that there exists a sufficiently small T^* such that

$$\|\hat{n}(x,T)\|_{L^2(\Omega)} + \|\hat{p}(x,T)\|_{L^2(\Omega)} = 0,$$

if $T \in [0, T^*]$. Then it follows that $\|\hat{f}\|_{L^2(\Omega)} = 0$ by (53), i.e. $\hat{f}(x) = 0$, a.e. in Ω. Corresponding to such $\hat{f}$, we deduce that the unique solution $(\hat{\psi}, \hat{n}, \hat{p})$ of the direct problem (50)-(52) and (54) is the trivial solution. That is,

$$\hat{\psi} = \hat{n} = \hat{p} = 0, \quad \text{a.e. in } Q_T.$$

So, the proof is completed. □

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Bin Wu,
Nanjing University of Information Science and Technology,
Nanjing 210044, China.
Email: wubing790831@126.com

Zewen Wang,
Department of Mathematics, School of Science,
East China Institute of Technology,
Nanchang 330013, China.
Email: wzwjskx@yahoo.com.cn