



Generalized System for Relaxed Cocoercive Mixed Variational Inequalities and Iterative Algorithms in Hilbert Spaces

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Abstract

The approximate solvability of a generalized system for relaxed cocoercive mixed variational inequality is studied by using the resolvent operator technique. The results presented in this paper extend and improve the main results of Chang et al.[1], He and Gu [2] and Verma [3, 4].

1 Introduction and Preliminaries

In this paper, the approximate solvability of a system of nonlinear variational inequalities involving two relaxed cocoercive mappings in Hilbert spaces is studied, based on the convergence of resolvent method.

Let H be a real Hilbert space, whose inner product and norm are denoted by $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$. Let I be the identity mapping on H , and $T(\cdot, \cdot), S(\cdot, \cdot): H \times H \rightarrow H$ be two nonlinear operator. Let $\partial\varphi$ denote the subdifferential of function φ , where $\varphi: H \rightarrow R \cup \{+\infty\}$ is a proper convex lower semicontinuous function on H . It is well known that the subdifferential $\partial\varphi$ is a maximal monotone operator. consider a systems of nonlinear variational inequalities (for short, SNVI) as follows: Find $x^*, y^* \in H$, such that

$$\langle \rho T(y^*, x^*) + x^* - y^*, x - x^* \rangle + \varphi(x) - \varphi(x^*) \geq 0, \forall x \in H, \rho > 0; \quad (1.1)$$

Key Words: relaxed cocoercive mixed variational inequality, resolvent method, relaxed cocoercive mapping, convergence of resolvent method.

2010 Mathematics Subject Classification: Primary 47H09; Secondary 47J05, 47J25.

Received: April, 2011.

Revised: April, 2011.

Accepted: February, 2012.

$$\langle \eta S(y^*, x^*) + y^* - x^*, x - y^* \rangle + \psi(x) - \psi(y^*) \geq 0, \forall x \in H, \eta > 0. \quad (1.2)$$

It is easy to know that the SNVI (1.1) and (1.2) is equivalent to the following projection equations:

$$\begin{aligned} x^* &= J_\varphi(x^* - \rho T(y^*, x^*)), \rho > 0; \\ y^* &= J_\psi(y^* - \eta S(x^*, y^*)), \eta > 0, \end{aligned}$$

where $J_\varphi = (I + \partial\varphi)^{-1}$, $J_\psi = (I + \partial\psi)^{-1}$.

Next we consider some special cases of the problem (1.1) and (1.2).

(I) If $T = S$, then the SNVI (1.1) and (1.2) reduces to the following system of nonlinear variational inequalities: find $x^*, y^* \in H$ such that

$$\langle \rho T(y^*, x^*) + x^* - y^*, x - x^* \rangle + \varphi(x) - \varphi(x^*) \geq 0, \forall x \in H, \rho > 0; \quad (1.3)$$

$$\langle \eta T(y^*, x^*) + y^* - x^*, x - y^* \rangle + \psi(x) - \psi(y^*) \geq 0, \forall x \in H, \eta > 0. \quad (1.4)$$

(II) If $\varphi = \psi$, then the SNVI (1.1) and (1.2) reduces to the following system of nonlinear variational inequalities: find $x^*, y^* \in H$ such that

$$\langle \rho T(y^*, x^*) + x^* - y^*, x - x^* \rangle + \varphi(x) - \varphi(x^*) \geq 0, \forall x \in H, \rho > 0; \quad (1.5)$$

$$\langle \eta S(y^*, x^*) + y^* - x^*, x - y^* \rangle + \varphi(x) - \varphi(y^*) \geq 0, \forall x \in H, \eta > 0. \quad (1.6)$$

(III) If $T = S$, $\varphi = \psi$, then the SNVI (1.1) and (1.2) reduces to the following system of nonlinear variational inequalities: find $x^*, y^* \in H$ such that

$$\langle \rho T(y^*, x^*) + x^* - y^*, x - x^* \rangle + \varphi(x) - \varphi(x^*) \geq 0, \forall x \in H, \rho > 0; \quad (1.7)$$

$$\langle \eta T(y^*, x^*) + y^* - x^*, x - y^* \rangle + \varphi(x) - \varphi(y^*) \geq 0, \forall x \in H, \eta > 0. \quad (1.8)$$

which was studied by He and Gu in [2].

(IV) If K is closed convex set in H , $\psi = \varphi$ and $\varphi(x) = I_K(x)$ for all $x \in K$, where I_K is the indicator function of K defined by $I_K(x) = \begin{cases} 0, & x \in K \\ +\infty, & \text{otherwise} \end{cases}$, then the SNVI (1.7) and (1.8) is equivalent to the following SNVI: find $x^*, y^* \in K$ such that

$$\langle \rho T(y^*, x^*) + x^* - y^*, x - x^* \rangle \geq 0, \forall x \in K, \rho > 0; \quad (1.9)$$

$$\langle \eta T(y^*, x^*) + y^* - x^*, x - y^* \rangle \geq 0, \forall x \in K, \eta > 0. \quad (1.10)$$

The problem (1.9) and (1.10) have been studied by Chang et al. (see [1]).

(V) If $T, S : H \rightarrow H$ are univariate mappings, then the SNVI (1.1) and (1.2) is collapsed to the following SNVI: find $x^*, y^* \in H$ such that

$$\langle \rho T(y^*) + x^* - y^*, x - x^* \rangle + \varphi(x) - \varphi(x^*) \geq 0, \forall x \in H, \rho > 0; \quad (1.11)$$

$$\langle \eta S(x^*) + y^* - x^*, x - y^* \rangle + \psi(x) - \psi(y^*) \geq 0, \forall x \in H, \eta > 0. \quad (1.12)$$

Further, if K is closed convex set in H , $S = T$, $\psi = \varphi$ and $\varphi(x) = I_K(x)$ for all $x \in K$, where I_K is the indicator function of K , then the SNVI (1.11) and (1.12) is equivalent to the following SNVI: find $x^*, y^* \in K$ such that

$$\begin{aligned} \langle \rho T(y^*) + x^* - y^*, x - x^* \rangle &\geq 0, \forall x \in K, \rho > 0; \\ \langle \eta T(x^*) + y^* - x^*, x - y^* \rangle &\geq 0, \forall x \in K, \eta > 0, \end{aligned}$$

which was studied by Verma in [3].

The following definitions and lemma are needed in the sequel.

Definition 1.1.

(i) A mapping $T : H \rightarrow H$ is called r -strongly monotone, if for each $x, y \in H$, we have

$$\langle T(x) - T(y), x - y \rangle \geq r \|x - y\|^2,$$

for a constant $r > 0$. This implies that

$$\|Tx - Ty\| \geq r \|x - y\|,$$

that is, T is r -expansive and when $r = 1$, it is expansive.

(ii) A mapping $T : H \rightarrow H$ is called μ -cocoercive, if there exists a constant $\mu > 0$ such that

$$\langle T(x) - T(y), x - y \rangle \geq \mu \|T(x) - T(y)\|^2, \forall x, y \in H.$$

Clearly, every μ -cocoercive mapping T is $\frac{1}{\mu}$ - Lipschitz continuous.

(iii) A mapping $T : H \rightarrow H$ is said to relaxed γ -cocoercive, if there exists a constant $\gamma > 0$ such that

$$\langle T(x) - T(y), x - y \rangle \geq -\gamma \|T(x) - T(y)\|^2.$$

(iv) $T : H \rightarrow H$ is said to be relaxed (γ, r) -cocoercive, if there exists constants $\gamma, r > 0$ such that

$$\langle T(x) - T(y), x - y \rangle \geq -\gamma \|T(x) - T(y)\|^2 + r \|x - y\|^2, \forall x, y \in H.$$

Remark 1.1. It follows from the above definitions that a r -strongly monotone mapping must be a relaxed (γ, r) -cocoercive mapping for $\gamma = 0$, but the converse is not true. therefore the class of the relaxed (γ, r) -cocoercive mappings is more general class.

Definition 1.2.

(1) A two-variable mapping $T : H \times H \rightarrow H$ is said to be relaxed (γ, r) -cocoercive, if there exist constant $\gamma, r > 0$ such that

$$\langle T(x, u) - T(y, v), x - y \rangle \geq -\gamma \|T(x, u) - T(y, v)\|^2 + r \|x - y\|^2, \forall x, y, u, v \in H.$$

(2) A mapping $T : H \times H \rightarrow H$ is said to be μ -Lipschitz continuous in the first variable, if there exists a constant $\mu > 0$ such that

$$\|T(x, u) - T(y, v)\| \leq \mu \|x - y\|, \forall x, y, u, v \in H.$$

Lemma 1.1. Suppose that $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ are nonnegative sequence satisfying the following inequality

$$a_{n+1} \leq (1 - t_n) a_n + b_n + c_n, n \geq 0,$$

where $t_n \in (0, 1)$, $\sum_{n=0}^{\infty} t_n = \infty$, $b_n = o(t_n)$, $\sum_{n=0}^{\infty} c_n < \infty$, then $\lim_{n \rightarrow \infty} a_n = 0$.

2. Algorithms

In this section, the general two-step models for approximate solutions to the SNVI (1.1) and (1.2) are given.

Algorithm 2.1. For arbitrary chosen initial points $x_0, y_0 \in H$ compute the sequences $\{x_n\}$ and $\{y_n\}$ such that

$$\begin{cases} x_{n+1} = (1 - \alpha_n - \delta_n) x_n + \alpha_n J_\varphi(y_n - \rho T(y_n, x_n)) + \delta_n u_n \\ y_n = (1 - \beta_n - \lambda_n) x_n + \beta_n J_\psi(x_n - \eta S(x_n, y_n)) + \lambda_n v_n, \end{cases} \quad (2.1)$$

where $J_\varphi = (I + \partial\varphi)^{-1}$, $J_\psi = (I + \partial\psi)^{-1}$, ρ and $\eta > 0$ are constants and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\lambda_n\}$, $\{\delta_n\}$ are sequences in $[0, 1]$ and $\{u_n\}$, $\{v_n\}$ are bounded sequences in H .

If $S = T$, then Algorithm 2.1 is reduced to the following:

Algorithm 2.2. For arbitrary chosen initial points $x_0, y_0 \in H$ compute the sequences $\{x_n\}$ and $\{y_n\}$ such that

$$\begin{cases} x_{n+1} = (1 - \alpha_n - \delta_n) x_n + \alpha_n J_\varphi(y_n - \rho T(y_n, x_n)) + \delta_n u_n \\ y_n = (1 - \beta_n - \lambda_n) x_n + \beta_n J_\psi(x_n - \eta T(x_n, y_n)) + \lambda_n v_n, \end{cases}$$

where $J_\varphi = (I + \partial\varphi)^{-1}$, $J_\psi = (I + \partial\psi)^{-1}$, ρ and $\eta > 0$ are constants and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\lambda_n\}$, $\{\delta_n\}$ are sequences in $[0, 1]$ and $\{u_n\}$, $\{v_n\}$ are bounded sequences in H .

If $\psi = \varphi$, then Algorithm 2.1 is reduced to the following:

Algorithm 2.3. For arbitrary chosen initial points $x_0, y_0 \in H$ compute the sequences $\{x_n\}$ and $\{y_n\}$ such that

$$\begin{cases} x_{n+1} = (1 - \alpha_n - \delta_n) x_n + \alpha_n J_\varphi(y_n - \rho T(y_n, x_n)) + \delta_n u_n \\ y_n = (1 - \beta_n - \lambda_n) x_n + \beta_n J_\varphi(x_n - \eta S(x_n, y_n)) + \lambda_n v_n, \end{cases}$$

where $J_\varphi = (I + \partial\varphi)^{-1}$, ρ and $\eta > 0$ are constants and $\{\alpha_n\}, \{\beta_n\}, \{\lambda_n\}, \{\delta_n\}$ are sequences in $[0, 1]$ and $\{u_n\}, \{v_n\}$ are bounded sequences in H .

If $S = T, \psi = \varphi$, then Algorithm 2.1 is reduced to the following:

Algorithm 2.4. For arbitrary chosen initial points $x_0, y_0 \in H$ compute the sequences $\{x_n\}$ and $\{y_n\}$ such that

$$\begin{cases} x_{n+1} = (1 - \alpha_n - \delta_n) x_n + \alpha_n J_\varphi(y_n - \rho T(y_n, x_n)) + \delta_n u_n \\ y_n = (1 - \beta_n - \lambda_n) x_n + \beta_n J_\varphi(x_n - \eta T(x_n, y_n)) + \lambda_n v_n, \end{cases}$$

where $J_\varphi = (I + \partial\varphi)^{-1}$, ρ and $\eta > 0$ are constants and $\{\alpha_n\}, \{\beta_n\}, \{\lambda_n\}, \{\delta_n\}$ are sequences in $[0, 1]$ and $\{u_n\}, \{v_n\}$ are bounded sequences in H .

3. Main Results

Based on Algorithm 2.1, the approximation solvability of the SNVI (1.1) and (1.2) is presented.

Theorem 3.1. Let H be a real Hilbert spaces. Let $T(\cdot, \cdot) : H \times H \rightarrow H$ be two-variable relaxed (γ_1, r_1) -cocoercive and μ_1 -Lipschitz continuous in the first variable; $S(\cdot, \cdot) : H \times H \rightarrow H$ be two-variable relaxed (γ_2, r_2) -cocoercive and μ_2 -Lipschitz continuous in the first variable. Suppose that $(x^*, y^*) \in H \times H$ is a solution of the problem (1.1) and (1.2) and that $\{x_n\}, \{y_n\}$ are the sequences generated by Algorithm 2.1. If $\{\alpha_n\}, \{\beta_n\}, \{\lambda_n\}$ and $\{\delta_n\}$ are four sequences in $[0, 1]$ satisfying the following conditions

- (i) $\sum_{n=0}^{\infty} \alpha_n = \infty, \sum_{n=0}^{\infty} \delta_n < \infty,$
- (ii) $\lim_{n \rightarrow \infty} (1 - \beta_n) = 0, \lambda_n = o(\alpha_n),$
- (iii) $0 < \rho < \frac{2(r_1 - \gamma_1 \mu_1^2)}{\mu_1^2}, 0 < \eta < \frac{2(r_2 - \gamma_2 \mu_2^2)}{\mu_2^2},$
- (iv) $r_i > \gamma_i \mu_i^2, i = 1, 2,$ then the sequences $\{x_n\}$ and $\{y_n\}$ converges strongly to x^* and y^* , respectively.

Proof. Since x^* and y^* are a solution to the SNVI (1.1) and (1.2), then

$$\begin{aligned} x^* &= J_\varphi(x^* - \rho T(y^*, x^*)), \rho > 0; \\ y^* &= J_\psi(y^* - \eta S(x^*, y^*)), \eta > 0, \end{aligned}$$

It follows from (2.1) that

$$\begin{aligned} \|x_{n+1} - x^*\| &= \|(1 - \alpha_n - \delta_n)x_n + \alpha_n J_\varphi(y_n - \rho T(y_n, x_n)) \\ &\quad - (1 - \alpha_n - \delta_n)x^* - \alpha_n J_\varphi(y^* - \rho T(y^*, x^*)) + \delta_n u_n - x^* \delta_n\| \\ &\leq (1 - \alpha_n - \delta_n)\|x_n - x^*\| + \delta_n(\|u_n\| + \|x^*\|) \\ &\quad + \alpha_n \|y_n - y^* - \rho(T(y_n, x_n) - T(y^*, x^*))\|. \end{aligned} \quad (3.1)$$

From the relaxed (γ_1, r_1) cocoercive and μ_1 -Lipschitz continuity in the first variable on T , we have

$$\begin{aligned} &\|y_n - y^* - \rho(T(y_n, x_n) - T(y^*, x^*))\|^2 \\ &= \|y_n - y^*\|^2 - 2\rho \langle T(y_n, x_n) - T(y^*, x^*), y_n - y^* \rangle \\ &\quad + \rho^2 \|T(y_n, x_n) - T(y^*, x^*)\|^2 \\ &\leq \|y_n - y^*\|^2 + \rho^2 \mu_1^2 \|y_n - y^*\|^2 - 2\rho r_1 \|y_n - y^*\|^2 \\ &\quad + 2\rho \gamma_1 \|T(y_n, x_n) - T(y^*, x^*)\|^2 \\ &\leq (1 + \rho^2 \mu_1^2 - 2\rho r_1 + 2\rho \gamma_1 \mu_1^2) \|y_n - y^*\|^2. \end{aligned} \quad (3.2)$$

Substituting (3.2) into (3.1) and simplifying the result, we have

$$\|x_{n+1} - x^*\| = (1 - \alpha_n - \delta_n) \|x_n - x^*\| + \theta_1 \alpha_n \|y_n - y^*\| + \delta_n (\|u_n\| + \|x^*\|). \quad (3.3)$$

where $\theta_1 = \sqrt{1 + \rho^2 \mu_1^2 - 2\rho r_1 + 2\rho \gamma_1 \mu_1^2} < 1$ by Condition (iii).

Now we make an estimation for $\|y_n - y^*\|$. It follows from (2.1) that

$$\begin{aligned} &\|y_n - y^*\| \\ &= \|(1 - \beta_n - \lambda_n)x_n + \beta_n J_\psi(x_n - \eta S(x_n, y_n)) \\ &\quad - (1 - \beta_n - \lambda_n)y^* - \beta_n J_\psi(y^* - \eta S(x^*, y^*)) + \lambda_n v_n - y^* \lambda_n\| \\ &\leq (1 - \beta_n - \lambda_n)\|x_n - y^*\| + \beta_n \|x_n - x^* - \eta[S(x_n, y_n) - S(x^*, y^*)]\| \\ &\quad + \lambda_n (\|v_n\| + \|y^*\|) \\ &\leq (1 - \beta_n - \lambda_n)\|x_n - x^*\| + (1 - \beta_n - \lambda_n)\|x^* - y^*\| \\ &\quad + \beta_n \|x_n - x^* - \eta[S(x_n, y_n) - S(x^*, y^*)]\| + \lambda_n (\|v_n\| + \|y^*\|). \end{aligned} \quad (3.4)$$

Next we estimate $\|x_n - x^* - \eta[S(x_n, y_n) - S(x^*, y^*)]\|$. From the relaxed (γ_2, r_2) cocoercive and μ_2 -Lipschitz cocoercive in the first variable on S , we

get

$$\begin{aligned}
 & \|x_n - x^* - \eta[S(x_n, y_n) - S(x^*, y^*)]\|^2 \\
 &= \|x_n - x^*\|^2 - 2\eta\langle S(x_n, y_n) - S(x^*, y^*), x_n - x^* \rangle \\
 &+ \eta^2\|S(x_n, y_n) - S(x^*, y^*)\|^2 \\
 &\leq \|x_n - x^*\|^2 + \eta^2\mu_2^2\|x_n - x^*\|^2 + 2\eta\gamma_2\|S(x_n, y_n) - S(x^*, y^*)\|^2 \\
 &- 2\eta r_2\|x_n - x^*\|^2 \\
 &\leq (1 + \eta^2\gamma_2^2 - 2\eta r_2 + 2\eta\gamma_2\mu_2^2)\|x_n - x^*\|^2.
 \end{aligned} \tag{3.5}$$

Let $\theta_2 = \sqrt{1 + \eta^2\gamma_2^2 - 2\eta r_2 + 2\eta\gamma_2\mu_2^2} < 1$ by Condition (iii). Substituting (3.5) into (3.4), we have

$$\begin{aligned}
 \|y_n - y^*\| &\leq (1 - \beta_n - \lambda_n)\|x_n - x^*\| + (1 - \beta_n - \lambda_n)\|x^* - y^*\| \\
 &+ \beta_n\theta_2\|x_n - x^*\| + \lambda_n(\|v_n\| + \|y^*\|).
 \end{aligned} \tag{3.6}$$

Combining (3.6) and (3.3), we obtain that

$$\begin{aligned}
 & \|x_{n+1} - x^*\| \\
 &= (1 - \alpha_n - \delta_n)\|x_n - x^*\| + \theta_1\alpha_n\|y_n - y^*\| + \delta_n(\|u_n\| + \|x^*\|) \\
 &\leq (1 - \alpha_n - \delta_n)\|x_n - x^*\| + \theta_1\alpha_n[(1 - \beta_n - \lambda_n)\|x_n - x^*\| \\
 &+ (1 - \beta_n - \lambda_n)\|x^* - y^*\| \\
 &+ \beta_n\theta_2\|x_n - x^*\| + \lambda_n(\|v_n\| + \|y^*\|)] + \delta_n(\|u_n\| + \|x^*\|) \\
 &\leq (1 - (1 - \theta_1)\alpha_n)\|x_n - x^*\| + \alpha_n[(1 - \beta_n - \lambda_n)\|x^* - y^*\| \\
 &+ \lambda_n(\|v_n\| + \|y^*\|)] + \delta_n(\|u_n\| + \|x^*\|).
 \end{aligned} \tag{3.7}$$

Set $a_n = \|x_n - x^*\|$, $t_n = (1 - \theta_1)\alpha_n$, $b_n = \alpha_n[(1 - \beta_n - \lambda_n)\|x^* - y^*\| + \lambda_n(\|v_n\| + \|y^*\|)]$ and $c_n = \delta_n(\|u_n\| + \|x^*\|)$ in (3.7). By Lemma 1.1 ensures that $x_n \rightarrow x^*$ as $n \rightarrow \infty$. This completes the proof.

Remark 3.2. Theorem 2.1 extends and improves the main results of [1], [2], [3] and [4], respectively.

The following theorems can be obtained from Theorem 3.1 immediately.

Theorem 3.3. Let H be a real Hilbert spaces. Let $T(\cdot, \cdot) : H \times H \rightarrow H$ be two-variable relaxed (γ_1, r_1) -cocoercive and μ_1 -Lipschitz continuous in the first variable. Suppose that $(x^*, y^*) \in H \times H$ is a solution of the problem (1.3) and (1.4) and that $\{x_n\}, \{y_n\}$ are the sequences generated by Algorithm 2.2. If $\{\alpha_n\}, \{\beta_n\}, \{\lambda_n\}$ and $\{\delta_n\}$ are four sequences in $[0, 1]$ satisfying the following conditions

$$\text{(i) } \sum_{n=0}^{\infty} \alpha_n = \infty, \quad \sum_{n=0}^{\infty} \delta_n < \infty,$$

- (ii) $\lim_{n \rightarrow \infty} (1 - \beta_n) = 0, \lambda_n = o(\alpha_n),$
 (iii) $0 < \rho < \frac{2(r_1 - \gamma_1 \mu_1^2)}{\mu_1^2}, 0 < \eta < \frac{2(r_2 - \gamma_2 \mu_2^2)}{\mu_2^2},$
 (iv) $r_i > \gamma_i \mu_i^2, i = 1, 2,$ then the sequences $\{x_n\}$ and $\{y_n\}$ converges strongly to x^* and y^* , respectively.

Theorem 3.4. Let H be a real Hilbert spaces. Let $T(\cdot, \cdot) : H \times H \rightarrow H$ be two-variable relaxed (γ_1, r_1) -cocoercive and μ_1 -Lipschitz continuous in the first variable; $S(\cdot, \cdot) : H \times H \rightarrow H$ be two-variable relaxed (γ_2, r_2) -cocoercive and μ_2 -Lipschitz continuous in the first variable. Suppose that $(x^*, y^*) \in H \times H$ is a solution of the problem (1.5) and (1.6) and that $\{x_n\}, \{y_n\}$ are the sequences generated by Algorithm 2.3. If $\{\alpha_n\}, \{\beta_n\}, \{\lambda_n\}$ and $\{\delta_n\}$ are four sequences in $[0, 1]$ satisfying the following conditions

- (i) $\sum_{n=0}^{\infty} \alpha_n = \infty, \sum_{n=0}^{\infty} \delta_n < \infty,$
 (ii) $\lim_{n \rightarrow \infty} (1 - \beta_n) = 0, \lambda_n = o(\alpha_n),$
 (iii) $0 < \rho < \frac{2(r_1 - \gamma_1 \mu_1^2)}{\mu_1^2}, 0 < \eta < \frac{2(r_2 - \gamma_2 \mu_2^2)}{\mu_2^2},$
 (iv) $r_i > \gamma_i \mu_i^2, i = 1, 2,$ then the sequences $\{x_n\}$ and $\{y_n\}$ converges strongly to x^* and y^* , respectively.

Theorem 3.5. Let H be a real Hilbert spaces. Let $T(\cdot, \cdot) : H \times H \rightarrow H$ be two-variable relaxed (γ_1, r_1) -cocoercive and μ_1 -Lipschitz continuous in the first variable. Suppose that $(x^*, y^*) \in H \times H$ is a solution of the problem (1.7) and (1.8) and that $\{x_n\}, \{y_n\}$ are the sequences generated by Algorithm 2.4. If $\{\alpha_n\}, \{\beta_n\}, \{\lambda_n\}$ and $\{\delta_n\}$ are four sequences in $[0, 1]$ satisfying the following conditions

- (i) $\sum_{n=0}^{\infty} \alpha_n = \infty, \sum_{n=0}^{\infty} \delta_n < \infty,$
 (ii) $\lim_{n \rightarrow \infty} (1 - \beta_n) = 0, \lambda_n = o(\alpha_n),$
 (iii) $0 < \rho < \frac{2(r_1 - \gamma_1 \mu_1^2)}{\mu_1^2}, 0 < \eta < \frac{2(r_2 - \gamma_2 \mu_2^2)}{\mu_2^2},$
 (iv) $r_i > \gamma_i \mu_i^2, i = 1, 2,$ then the sequences $\{x_n\}$ and $\{y_n\}$ converges strongly to x^* and y^* , respectively.

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