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A PDE based Model for Sonar Image and Video Denoising

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Abstract

A PDE model for reducing the speckle noise from ultrasound video sequences is proposed in this article. A spatial approach is used for video denoising. First, a continuous differential mathematical model for noise removal is proposed. Then, a discretized variant of the denoising approach is described. The proposed PDE based technique is successfully applied to sonar images and videos.

1 Introduction

We propose a partial differential equations based mathematical model for ultrasound video denoising in this paper. The video pre-processing operations are intended to enhance the ultrasound movie and to prepare it for a proper video analysis. Most important pre-processing tasks are related to video denoising and restoration.

The signals of sonar sequences are very often affected by noise. The most common type of noise in images and videos is the speckle noise. It represents a multiplicative noise, that is locally correlated and prevents a proper feature extraction and analysis from the affected video images [1].

Obviously, in these situations some noise filtering and signal restoration techniques are required. These operations must keep the ultrasound movie quality and do not destroy its interesting details [1,2].

The main drawback of the most image and video denoising methods is the blurring effect. The techniques based on mean and median filters [2] could delete the details and alter the feature extraction and video object detection

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processes. In recent years, some smoothing techniques which can solve this problem have been developed.

We can mention here the Frost filtering [2], that replaces the pixel of interest with the weighted sum of the values in a moving kernel, the Discrete Wavelet Transform based approaches, using complex DWT-2D and DWT-3D [3], and the differential equation based image and video denoising techniques [4].

Partial differential equations (PDE) are increasingly used in the image and video processing domains. Many image filtering techniques, based on second and fourth order PDE, have been proposed in the last decade [4]. We have already approached the PDE-based image processing domain, by developing a PDE variational technique to image denoising and restoration [5]. Now, we construct a PDE based method that is able to reduce this special type of noise (speckle) and works properly for ultrasound video sequences.

Video signal smoothing can be performed using three categories of filtering techniques: spatial, temporal and spatio-temporal approaches. The spatial methods perform video denoising by sequential filtering of the video frames. The temporal smoothing approaches use video motion estimation, while the spatio-temporal techniques represent combined methods.

We propose a spatial video denoising model in this paper. The continous mathematical model is described in the next section. The discretized version of this PDE model is presented in the third section. Some experiments are described in the fourth section. The work ends with a conclusions section and a list of references.

2 A continuous mathematical model for video smoothing

In this section we present the continuous form of our proposed PDE based noise removal model. It represents an image denoising mathematical model that is extended to be applied for sonar movies.

Therefore, let us consider an ultrasound video sequence, $V = \{I_1, ?, I_n\}$, that is affected by speckle noise. Each continuous image representing a video frame can be described as a binary function $I_i : R^2 \to R$. With these notations, the proposed PDE speckle denoising model is expressed by the following differential equation:

$$\frac{\partial I_i}{\partial t} = -\Delta[f(\Delta I_i)\Delta I_i], \quad \forall i \in [1, n], \tag{1}$$

where $\Delta I_i = \nabla^2 I_i$ and f represents a noise smoothing function that is modelled as in the following relation:

$$f(x) = \frac{x}{x^2 + k}, \tag{2}$$

where $k > 0$ represents a chosen constant. Obviously, this function f is monotone decreasing and converges to 0. Thus, it satisfies the main conditions which are essential for a noise filtering function [4]:

$$\begin{cases} \lim_{x \to \infty} f(x) = 0 \\ f(0) = 1 \end{cases} \tag{3}$$

Let us discuss now the well-posedness of the problem (1), because most PDE based denoising systems do not discuss it at all. We briefly show below that this problem is indeed well posed if the function $u \to f(u)u \equiv g(u)$ is continuous and monotonically nondecreasing [6]. To this end, we consider in the space $L^2(\Omega)$, where Ω is the image domain the nonlinear differential operator:

$$Au = \Delta g(\Delta u) \tag{4}$$

with the domain

$$D(A) = \left\{ u \in H^2(\Omega);\ g(\Delta u) \in H^2(\Omega),\ \frac{\partial u}{\partial n} = 0,\ \frac{\partial}{\partial n} g(\Delta u) = 0 \text{ on } \partial\Omega \right\}, \tag{5}$$

where $H^2(\Omega)$ is the Sobolev space of order 2. We have:

Proposition 1. *Assume that $g(u)u \geq \alpha u^2$, $\forall u \in R$. The operator A is maximal monotone in $L^2(\Omega)$, that is,*

$$\int_\Omega (Au - Av)(u - v)dx \geq 0, \quad \forall u, v \in D(A) \tag{6}$$

and the range $R(I + A)$ is $L^2(\Omega)$.

Proof. Inequality (6) follows by Green formula while equation $u + Au = v$; $v \in L^2(\Omega)$ reduces to the minimization problem

$$\text{Min} \left\{ \int_\Omega \left(j(\Delta u) + \frac{1}{2}(u - v) \right)^2 \right\} dx, \quad u \in H^2(\Omega), \tag{7}$$

where $j(y) = \int_0^y g?dr$, $\forall y \in R$.

On the other hand, since $r? \geq \alpha r^2$ for all r, it follows that problem (7) has a unique solution u. This completes the proof. Then, by the standard existence theory for nonlinear evolution of monotone type [6], it follows that the Cauchy problem

$$\frac{\partial u}{\partial t} + Au = 0, \quad t > 0; \ u(0) = u_0 \tag{8}$$

has a unique strong solution, that is $u = u(t)$. Moreover, it turns out that $\lim_{t\to\infty} u(t) = u^*$, where $Au^* \equiv 0$. This differential model for noise reducing is converted into the discrete form in the next section.

3 Discretized version of the PDE denoising approach

We consider a space grid of size μ and a time discretization of size h [5]. We take the size of image to be $[N\mu \times M\mu]$ and the time interval $T = Ph$. Then, we set

$$x_k = k\mu, \ y_j = j\mu, \ t_\ell = \ell h, \tag{9}$$

for $k = 0, 1, ..., N; \ j = 0, 1, ..., M; \ \ell = 0, 1, ..., P$; and denote by $(I_i)_{k,j}^{\ell}$ the value of I_i at the point (t_ℓ, x_k, y_j), that is,

$$(I_i)_{k,j}^{\ell} = I_i(t_\ell, x_k, y_j). \tag{10}$$

We discretize equation (1) with respect to t and (x, y) by replacing the Laplace operator Δ by its finite difference version:

$$\Delta_{k,j} u = \frac{1}{\mu^2} \left[u_{k+1,j} + u_{k-1,j} + u_{k,j+1} + u_{k,j-1} - 4u_{k,j}\right], \quad j < M, k < N \tag{11}$$

and first order differential operator $\dfrac{\partial}{\partial t}$ by

$$D^\ell u = \frac{1}{h} \left[u^{\ell+1} - u^\ell\right], \quad \ell = [0, P]. \tag{12}$$

To $\Delta_{k,j}$ we impose boundary value conditions of zero flux boundary type, which have the following form:

$$\begin{cases} u_{N+1,j} = u_{N,j}, & u_{-1,j} = u_{0,j}, \quad j = 0,...,M, \\ u_{k,-1} = u_{k,0}, & u_{k,M+1} = u_{k,M}, \quad k = 0,...,N. \end{cases} \tag{13}$$

We set $d_{k,j} = f(\Delta_{k,j}u)\Delta_{k,j}u$ and define the finite difference discretization of the operator $u \to \Delta f(\Delta u)\Delta u$ as follows:

$$\Delta_{k,j}(f(\Delta u)\Delta u) = \frac{1}{\mu^2}\,[d_{k+1,j} + d_{k-1,j} + d_{k,j+1} + d_{k,j-1} - 4d_{k,j}], \tag{14}$$

for $j < M,\ k < N$. Taking into account (12)-(14), we consider for equation (1) the following approximation scheme:

$$(I_i)^{\ell+1}_{k,j} = (I_i)^{\ell}_{k,j} + [d^{i,\ell}_{k-1,j} + d^{i,\ell}_{k,j+1} + d^{i,\ell}_{k,j-1} - 4d^{i,\ell}_{k,j}], \tag{15}$$

where $k \in [0, N],\ k \in [0, P],\ i \in [1, n]$. The boundary value conditions of the form (13) should be imposed to (15), therefore it results the following:

$$\begin{cases} (I_i)^{\ell}_{N+1,j} = (I_i)^{\ell}_{N,j} & (I_i)^{\ell}_{-1,j} = (I_i)^{\ell}_{0,j}, \quad j = 0,...,M, \\ (I_i)^{\ell}_{k,-1} = (I_i)^{\ell}_{k,0} & (I_i)^{\ell}_{k,M+1} = (I_i)^{\ell}_{k,M}, \quad k = 0,...,N, \end{cases} \tag{16}$$

and similar conditions for $d^{e,\ell}_{k,j} = f(\Delta_{k,j}I^{\ell}_i)\Delta_{k,j}I^{\ell}_i)$. Also, besides relations given by (16), we fix the initial value conditions $(I_i)^0_{k,j}$, where $j \le M$ and $k \le N$.

4 Experiments

We have performed a series of numerical experiments using the proposed PDE based denoising approach and obtained quite satisfactory results. The mathematical model has been successfully tested on various ultrasound image and video datasets. The speckle noise has been substantially reduced in hundreds of sonar images and video frames processed with the provided filter.

For example, in Figure 1 it is displayed a video frame from a radar movie, depicting a military vehicle that is moving on a battlefield. Obviously, the image is seriously affected by a great amount of speckle noise.

The ultrasound video affected by speckle is then filtered using the PDE based technique and the filtering result for the noised frame depicted in Figure 1 is displayed in Figure 2.

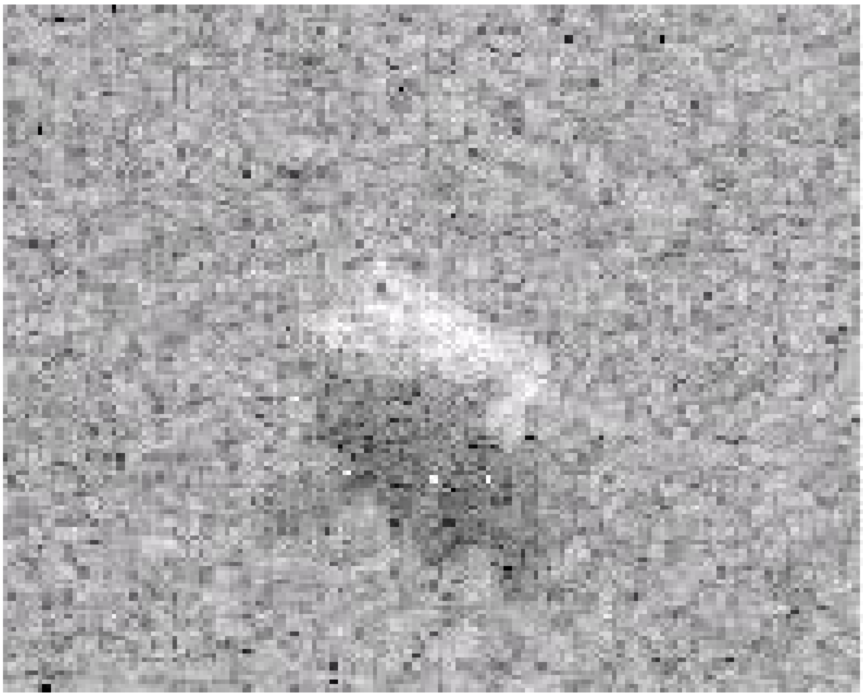

Figure 1: Video frame affected by speckle noise

One can see that the speckle noise have been reduced, although the operation have not performed a complete noise removal and some blurring efect is present. The video object of interest, the military vehicle, can be more easily visualized and detected in the second figure, this being the most important fact.

We have compared our PDE based filtering model with many other image and video denoising approaches. Our technique provides much better speckle noise removal results than Frost filters, mean or median filters, or the local region filters [1,2].

5 Conclusions

A mathematical model, based on partial differential equations, for sonar image and video smoothing has been provided on this paper. We have proposed a continuous PDE model, then discretized it to be implemented.

We have chosen a proper denoising function for our model and, what is

Figure 2: The video image filtering result: speckle noise reduced

more important, we have demonstrated the mathematical correctness of the proposed differential model. The developed technique has been successfully tested on video sequences affected by speckle noise, a visible noise reduction being achieved.

Although our method does not perform a total noise removal and the filtered image are affected by some blurring effect, it provides better results than other smoothing techniques. We intend to improve our denoising method and to make it applicable to other types of image noise too. Thus, in our future research on this image and video processing domain, we will try to combine this filtering model with other noise removal techniques [2], for enhancing its smoothing effectiveness.

Our video denoising technique represents a good pre-processing step for ultrasound video analysis. An efficient smoothing process facilitates the analysis and identification of the main video objects in the sonar sequence [7]. Our future research will focus also on video detection, tracking and motion estimation in sonar movies.

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