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LINEAR STABILITY RESULTS IN A MAGNETOTHERMOCONVECTION PROBLEM

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Abstract

An analytical study of a magnetothermoconvection problem where the exchange of stabilities holds and the dynamically free boundaries are thermally and electrically perfectly conducting is performed. The importance of the boundary conditions-independent part, hidden by the use of Fourier series methods, but evidentiated by the direct method based on the characteristic equation is shown. It is emphasized that the secular equation splitting provides a basis for extending the Chandrasekhar's power law [4] type results to a wider class of problems in linear stability of any continuum.

1 Introduction

Interactions of fluid flows with other phenomena such as magnetic or electric ones gave rise to adjacent domains, e.g. magnetohydrodynamics (MHD), electrohydrodynamics (EHD) or ferrohydrodynamics (FHD). Each of them has a large variety of applications and has been studied intensively. Most of the MHD problems governing the motion of a conducting fluid in a magnetic field concern liquid metals or plasma. The major use of MHD is in plasma physics, astrophysical plasma for instance. Another application of great interest is the generation of electricity by using liquid metals driven by a magnetic field.

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Time-independent MHD equilibria also form the basis for more advanced kinetic and particle-based models of plasma behaviour.

In Tokamak experiments, the so-called tearing instability, destroying the equilibrium magnetic configuration is observed. In [3] the existence of successive (bifurcation stationary Hopf) for the sets of magnetohydrodynamic equations generally depending on some radial coordinate, was proven mathematically.

The main purpose of a linear stability study and in particular of the stationary convection appearance is to find the critical values of the eigenvalue represented by the Rayleigh number for various values of the other physical parameters for which the instability sets in. In industrial processes this offers the possibility to eliminate the causes of the occurrence of turbulence by controlling the physical parameters, the shape or the machine design.

Herein the analytical investigation is performed for a magnetothermoconvection problem where the electrically perfectly conducting and dynamically free boundaries are thermally perfectly conducting and overstability is valid. This S. Chandrasekhar's problem (1952) has been investigated in [1] by using linear stability analysis proving the existence of overstable motions when the boundaries are dynamically free and thermally and electrically perfectly conducting. A complete proof of the Chandrasekhar prediction for situations of a quite general nature of the boundary surfaces was also yielded. The physical problem of modelling of the initiation of stationary magnetohydrodynamic convection manifesting in a simple Bénard configuration was initially investigated by Chandrasekhar. He proved that an asymptotic dependence of the critical Rayleigh number on the Chandrasekhar number Q takes place in the case of both boundary surfaces dynamically free and electrically conducting. Then the numerical evaluations for the case of both surfaces rigid and the case of one rigid and the other dynamically free led to the so-called Chandrasekhar conjecture: this type of power law $\pi^2 Q$ also holds.

The initiation of the magnetothermoconvection as solution bifurcating from the conduction state in a horizontal layer of a thermally and electrically conducting fluid is governed by the two-point problem

$$\begin{cases} (D^2-a^2)(D^2-a^2-p/\sigma)W = Ra^2\Theta - QD(D^2-a^2)h, \\ (D^2-a^2-p)\Theta = -W, \\ (D^2-a^2-p\sigma_1/\sigma)h = -DW, \end{cases} \tag{1}$$

$$W = \Theta = D^2W = h = 0 \text{ at } z = \pm 0.5 \tag{2}$$

where a represents the wavenumber, σ_1 and σ are the thermal and magnetic Prandtl number,respectively, R is the Rayleigh number, Q is the Chandrasekhar number, $p = p_r + ip_i$ is the complex growth rate of the normal mode perturbation, z is the independent variable in the vertical upwards direction, $D \equiv \frac{d}{dz}$, W is the vertical component of the velocity, Θ is the amplitude of the temperature perturbation field and h is the amplitude of the magnetic perturbation field, $W, \Theta, h : [-0.5, 0.5] \to \mathbb{C}$.

For the case of rigid boundaries surfaces the boundary conditions read

$$W = \Theta = DW = h = 0 \text{ at } z = \pm 0.5 \tag{3}$$

Further on we consider only the case $p = 0$, i.e. we assume that the principle of exchange of stabilities holds. Then the neutral stability of the thermal conduction is governed by the eigenvalue problem (1), (2) where $p = 0$, consisting of a system of linear ordinary differential equations (1) with constant coefficients and a set of boundary conditions (2). Here R is the eigenvalue and the vector of the unknown functions $\mathbf{U} = (W, \Theta, h)$ is the corresponding eigenvector. However, the study can be carried out for other stabilities on the same lines and for other continua too.

Problem (1)-(2) can be written in a matriceal form as:

$$\begin{cases} L\mathbf{U} = \mathbf{0} & -0.5 \le z \le 0.5, \\ B\mathbf{U} = \mathbf{0}, & z = \pm 0.5. \end{cases} \tag{4}$$

with

$$L = \begin{pmatrix} (D^2 - a^2)^2 & -Ra^2 & QD(D^2 - a^2) \\ 1 & D^2 - a^2 & 0 \\ D & 0 & D^2 - a^2 \end{pmatrix}$$

and the boundary conditions written for both dynamically free bounding surfaces. The domain of definition of the matriceal differential operator L is

$$\mathcal{D}(L) = \{\mathbf{U} = (W, \Theta, h) \in (L^2(-0.5, 0.5))^3 | W = D^2W = \Theta = h = 0 \text{ at } z = \pm 0.5\}.$$

Each unknown function in problem (1)-(2) can be uniquely written as a sum of an odd function and an even function, i.e. $W = W_o + W_e$, $\Theta = \Theta_0 + \Theta_e$, $h = h_e + h_o$. Since an even function is equal to an odd one only if they are both null, the problem splits in the following two-point problems

$$\begin{cases} (D^2 - a^2)^2 W_e = Ra^2 \Theta_e - QD(D^2 - a^2) h_o, \\ (D^2 - a^2)\Theta_e = -W_e, \\ (D^2 - a^2) h_e = -DW_o, \end{cases} \tag*{$(1)_e$}$$

$$W_e = D^2 W_e = \Theta_e = h_o = 0 \text{ at } z = \pm 0.5. \tag*{$(2)_e$}$$

and

$$\begin{cases} (D^2 - a^2)^2 W_o = Ra^2 \Theta_o - QD(D^2 - a^2) h_e, \\ (D^2 - a^2)\Theta_o = -W_o, \\ (D^2 - a^2) h_o = -DW_e, \end{cases} \tag*{$(1)_o$}$$

$$W_o = D^2 W_o = \Theta_o = h_e \text{ at } z = \pm 0.5. \qquad (2)_o$$

In spite of the fact that the functions W_e, Θ_e, occurring in $(1)_e$ - $(2)_e$ are even and h_o is odd, each term in equation $(1)_e$ is an even function. This is the reason why $(1)_e$-$(2)_2$ is called the even problem. Similarly, $(1)_o$ - $(2)_o$ is called the odd problem. Correspondingly, the general secular equation SE_g can be written as $SE_g = SE_e \cdot SE_o$, where $SE_e = 0$ and $SE_o = 0$ are the equations of the neutral hypersurface corresponding to even and odd problems respectively. Therefore solving the eigenvalue problem (1)-(2) is equivalent to solving problems $(1)_e$-$(2)_e$ and $(1)_o$-$(2)_o$ in appropriate spaces of even and odd functions separately. The smallest eigenvalue R_{min} will be a solution of $SE_e = 0$ or $SE_o = 0$ and, as a consequence, the corresponding eigenfunctions W, Θ, Dh will be even or odd.

Remark that, if for a particular choice of the parameters, the smallest eigenvalue corresponds to the even solution, then the situation still holds for any other values of those parameters [6].

Obviously, by solving $SE_e = 0$ or $SE_o = 0$ instead of $SE_g = 0$ simplifies the computations, i.e. instead of evaluating a n-th order determinant, we shall evaluate an $\frac{n}{2}$-th order determinant [8].

In Section 2.1 we present the results from [1], in Section 2.2 we apply the Fourier series techniques and in Section 2.3 a direct method based on the characteristic equation. All these approaches show that the secular equation can be written as a product of two terms: one taking into account the boundary conditions and the other leading to the determination of the smallest eigenvalue from the characteristic equation.

2 Analytical methods

The analytical methods used by us in order to solve (1)-(2) are: the Budianski-DiPrima (B-D) method, the Chandrasekhar-Galerkin method (C-G) and the direct method. The first two methods are based on Fourier series expansions. In the B-D method and its variant, used in [1] by Banerjee, Shandil and Kumar (B-Sh-K method) and presented by us, the eigensolution is looked for in the form of a Fourier series whose expansion functions do not satisfy all the boundary conditions of the eigenfunctions. Each boundary condition which is not fulfilled will introduce a constraint in the form of a series involving the Fourier coefficients. The Fourier coefficients are solutions of a system of linear nonhomogeneous algebraic equations obtained by imposing to the Fourier expansion of the eigensolution to satisfy the boundary conditions. In the B-D method the Fourier coefficients are deduced from the system in terms of some unknown boundary values of the eigenfunctions and/or their derivatives defining the constraints. Then they are introduced into the constraints to produce a secular equation in the form of an infinite series. In the C-G method the secular equation is an infinite determinant.

2.1 The B-Sh-K method

In the B-Sh-K method the Fourier coefficients of all eigenfunctions are expressed in terms of the Fourier coefficients of one eigenfunction alone (this is equivalent to solving the algebraic system). Next the constraints are used as another algebraic system which provides some of the Fourier coefficients in terms of others. Thus, the corresponding eigenfunctions satisfy all the boundary conditions. Then the application of the Galerkin-Ritz methods yields the secular equation.

Following [2] let us consider the set $\{\cos(\pi z), \cos(3\pi z), \cos(5\pi z), ...\}$ total in the subspace of $L^2(-0.5, 0.5)$ consisting of even functions.

$\{\sin(\pi z), \sin(3\pi z), \sin(5\pi z), ...\}$ is a total set in the subspace of $L^2(-0.5, 0.5)$ consisting of odd functions. In the case of free boundaries, we assume that h is even and that Θ, W are odd, $h = \sum_{n=0}^{\infty} h_n \cos(2n+1)\pi z$, where the corresponding boundary conditions (2) are taken into account, and

$$\Theta(z) = \sum_{n=0}^{\infty} \Theta_n \sin(2n+1)\pi z, W(z) = \sum_{n=0}^{\infty} W_n \sin(2n+1)\pi z.$$

Using the backward integration technique and introducing the notation $DW(0.5) = \alpha$, $D^3W(0.5) = \beta$, $D\Theta(0.5) = \gamma$, $(1)_{2,3}$ imply

$$W_n = E_n h_n, \quad \Theta_n = \frac{h_n}{(2n+1)\pi} + \frac{2\sqrt{2}(-1)^n\gamma}{E_{2n+1}(2n+1)\pi} \tag{5}$$

where $E_n = \dfrac{[(2n+1)^2\pi^2 + a^2]}{(2n+1)\pi}$. An equivalent expression for Θ_n can be derived from $(1)_1$ in the form

$$\Theta_n = \frac{1}{Ra^2}\{(E_n^2+a)(2n+1)^2\pi^2 E_n h_n + 2\sqrt{2}(-1)^{n-1}[\beta+2a^2\gamma] - (2n+1)^2\pi^2\alpha\}. \tag{6}$$

Elimination of h and Θ between $(1)_{1,2,3}$ leads to the equation

$$[(D^2 - a^2)^3 + Ra^2 - QD^2(D^2 - a^2)]W = 0. \tag{7}$$

Taking into account (2) an additional boundary condition can be considered, i.e. $D^4W = 0$. Three constraints must be imposed

$$\sum_{n=0}^{\infty}(-1)^n E_n h_n = 0, \tag{8}$$

$$\sum_{n=0}^{\infty}(-1)^n\{-(2n+1)^2\pi^2 E_n h_n + 2\alpha\} = 0, \tag{9}$$

$$\sum_{n=0}^{\infty}(-1)^n\{J_n E_n h_n + 2[\beta-(2n+1)^2\pi^2\alpha]\} = 0, \tag{10}$$

where $J_n = (2n+1)^4\pi^4$. From (8)-(10) the expressions of h_1, h_2 and h_3 in terms of h_0, h_4, h_5,.. can be deduced. We have

$$h_1 = \frac{9}{5}\frac{E_0}{E_1}h_0 + \frac{14}{5}\frac{E_4}{E_1}h_4 + ... + (-1)^n\frac{\{[(2n+1)^2-35]-4(2n+1)^2\}}{640}\frac{E_n}{E_1}h_n + ..., \tag{11}$$

$$h_2 = \frac{E_0}{E_2}h_0 + 6\frac{E_4}{E_2}h_4 + ... + (-1)^n\frac{\{[(2n+1)^2-21]^2-16(2n+1)^2\}}{384}\frac{E_n}{E_2}h_n + ..., \tag{12}$$

$$h_3 = \frac{1}{5}\frac{E_0}{E_3}h_0 + \frac{21}{5}\frac{E_4}{E_3}h_4 + ... + (-1)^n\frac{\{[(2n+1)^2-15]^2-4(2n+1)^2\}}{960}\frac{E_n}{E_3}h_n + ... \tag{13}$$

Introducing these expressions in the Fourier series for h, W and Θ we obtain expansions which satisfy all the boundary conditions (2) although these conditions are not satisfied for each expansion function. The expression for W reads

$$W(z) = E_0h_0\sin\pi z + \Big(\frac{9}{5}E_0h_0 + \frac{14}{5}E_4h_4 - \frac{54}{5}E_5h_5 + 27E_6h_6 + ... +$$

$$+(-1)^n\frac{\{[(2n+1)^2-35]^2-4(2n+1)^2\}}{640}E_nh_n + ...)\sin 3\pi z + (E_0h_0 + 6E_4h_4 -$$

$$-21E_5h_5 + E_6h_6 + ... + (-1)^n\frac{\{[(2n+1)^2-21]^2-16(2n+1)^2\}}{384}E_nh_n + ...\Big)$$

$$\sin 5\pi z + (\frac{1}{5}E_0h_0 + \frac{21}{5}E_4h_4 - \frac{56}{5}E_5h_5 + E_6h_6 + ... +$$

$$+(-1)^n\frac{\{[(2n+1)^2-15]^2-4(2n+1)^2\}}{960}\cdot E_nh_n + ...)\sin 7\pi z +$$

$$+\sum_{n=4}^{\infty}E_nh_n\sin(2n+1)\pi z$$

and, taking into account (7), leads to the following secular equation

$$\begin{vmatrix} L_0 & 0 & 0 & 0 & \dots & 0 & \dots \\ \frac{9}{5}L_1 & \frac{14}{5}L_1 & -\frac{54}{5}L_1 & 27L_1 & \dots & (-1)^n \frac{\{[(2n+1)^2-35]^2-4(2n+1)^2\}}{640}L_1 & \dots \\ L_2 & 6L_2 & -21L_2 & 50L_2 & \dots & (-1)^n \frac{\{[(2n+1)^2-21]^2-16(2n+1)^2\}}{384}L_2 & \dots \\ \frac{1}{5}L_3 & \frac{21}{5}L_3 & -\frac{56}{5}L_3 & 24L_3 & \dots & (-1)^n \frac{\{[(2n+1)^2-15]^2-4(2n+1)^2\}}{960}L_3 & \dots \\ 0 & L_4 & 0 & 0 & \dots & 0 & \dots \\ 0 & 0 & L_5 & 0 & \dots & 0 & \dots \end{vmatrix} = 0$$

where $L_k = [(2k+1)^2\pi^2 + a^2]^4 - Ra^2[(2k+1)^2\pi^2 + a^2] + Q[(2k+1)^2\pi^2 + a^2]^2$.

2.2 The B-D method

In the Hilbert space $L^2(-0.5, 0.5)$ we consider the bases $\{E_{2n-1}\}_{n\in\mathbb{N}}$, $E_{2n-1}(z) = \sqrt{2}\cos(2n-1)\pi z$ and $\{F_{2n-1}\}_{n\in\mathbb{N}}$, $F_{2n-1}(z) = \sqrt{2}\sin(2n-1)\pi z$. The B-D method is a weighted residual type method so that, taking into account also the parity of the unknown functions in (4), the solution has the spectral representation

$$W = \sum_{n=1}^{\infty} W_{2n-1}E_{2n-1}(z),\ \Theta = \sum_{n=1}^{\infty} \Theta_{2n-1}E_{2n-1}(z),\ h = \sum_{n=1}^{\infty} h_{2n-1}F_{2n-1}(z). \tag{14}$$

The boundary conditions $(2)_o$ for W and Θ are automatically satisfied, while the condition for h_e, i.e. $h_e(\pm 0.5) = 0$, introduces a constraint.

The series expansions of the derivatives occurring in $(1)_o$ are obtained by the backward integration technique [8]. Substitute these expressions in $(1)_o$ and impose the condition that the obtained equations be orthogonal to E_{2m-1}, $m = 1, 2, ...$ to get the system

$$\begin{cases} A_n^2 W_{2n-1} - Ra^2\Theta_{2n-1} - Q(2n-1)\pi A_n h_{2n-1} = 2\sqrt{2}(-1)^n\alpha_1(2n-1)\pi Q, \\ W_{2n-1} - A_n\Theta_{2n-1} = 0, \\ -(2n-1)\pi W_{2n-1} - A_n h_{2n-1} = 2\sqrt{2}\alpha_1(-1)^n, \end{cases}$$

with the notation $\alpha_1 = Dh(0.5)$ and $A_n = (2n-1)^2\pi^2 + a^2$. The secular equation has the form

$$\sum_{n=1}^{\infty} 4\alpha_1 \frac{A_n^3 + (2n-1)^2\pi^2 Q A_n - Ra^2}{A_n^4 + (2n-1)^2\pi^2 A_n^2 Q - Ra^2 A_n} = 0. \tag{15}$$

If $A_n^3 + (2n-1)^2\pi^2 Q A_n - Ra^2 \neq 0$ the secular equation is equivalent to the impossible relation $\sum\limits_{n=1}^{\infty} \dfrac{1}{A_n} = 0$. Therefore we must have

$$R = \frac{A_1^3 + \pi^2 Q A_1}{a^2} = \frac{(\pi^2 + a^2)[(\pi^2 + a^2)^2 + \pi^2 Q]}{a^2}.$$

Let us now apply the C-G method. First we modify the system by a translation of the variable z, $x = z + \dfrac{1}{2}$ allowed by the symmetry of the eigenvalue problem with respect to $z = 0.5$. Then the boundary conditions can be written at 0 and 1. Following the strategy from [10] we recast the eigenvalue problem (1)-(2) in a system of differential equations supplied with boundary conditions only and no constraints. Using the notation $T = (D^2 - a^2)W$ we have

$$\begin{cases} (D^2 - a^2)T - Ra^2\Theta + QD(D^2 - a^2)h = 0, \\ (D^2 - a^2)\Theta + W = 0, \\ (D^2 - a^2)h + DW = 0, \\ (D^2 - a^2)W - T = 0, \end{cases} \tag{16}$$

with the boundary conditions

$$T = W = \Theta = h = 0 \text{ at } z = 0, 1. \tag{17}$$

In this case a suitable spectral representation is based on the complete set of functions $\{\sin(n\pi z)\}_{n\in\mathbb{N}^*}$, $\{\cos(n\pi z)\}_{n\in\mathbb{N}^*}$ from $L^2(0,1)$, i.e.

$$T = \sum_{n=1}^{\infty} T_n \sin(n\pi z),\ W = \sum_{n=1}^{\infty} W_n \sin(n\pi z),\ \Theta = \sum_{n=1}^{\infty} \Theta_n \sin(n\pi z),$$

$$h = \sum_{n=1}^{\infty} h_n \cos(n\pi z).$$

The corresponding infinite algebraic system for the Fourier coefficients T_n, W_n, Θ_n, h_n, $n \in \mathbb{N}^*$ has the form

$$\begin{cases} -B_n T_n - Ra^2\Theta_n + Qn\pi h_n[B_n + a^2] = -Qn\pi\alpha', \\ -B_n\Theta_n + W_n = 0, \\ -B_n h_n + n\pi W_n = \alpha', \\ -B_n W_n - T_n = 0, \end{cases} \tag{18}$$

with the notations $\alpha' = \alpha_2(-1)^{n-1} + \beta_2$, $\alpha_2 = Dh(1)$, $\beta_2 = Dh(0)$. Then the secular equation becomes $\sum\limits_{n=1}^{\infty}(-1)^n\alpha' \dfrac{B_n^2 + QB_n n^2\pi^2 - Ra^2}{B_n^3 + QB_n n^2\pi^2(B_n + a^2) - Ra^2 B_n} = 0$ and it has no solution for $B_n^2 + QB_n n^2\pi^2 - Ra^2 \neq 0$ since it reduces to $\sum\limits_{n=1}^{\infty}\dfrac{1}{B_n} = 0$, with $B_n = n^2\pi^2 + a^2$.

However, if $B_n^2 + QB_n n^2\pi^2 - Ra^2 = 0$, then the same neutral curve as for the secular equation (15) follows.

2.3 The direct method

So far, in the free boundaries case, the problem was solved using methods based on Fourier series expansions satisfying all or just a part of the boundary conditions and the idea that in the general case the secular equation does not have any solution was emphasized. Herein the main idea is pointed out: the independence of the eigenvalues of the boundary conditions.

This possibility was found already in [7] and then in [9]. More precisely, in [7] by using the method based on the characteristic equation, three concrete problems on fluid flow stability were studied. In our case, it was found that the secular equation, yielding the eigenvalues splitted into a product of two equations: one depending and the other not depending on the boundary conditions. The boundary conditions-dependent equation had no solution while the other led to

$$\prod_{k=1}^{3}\cosh(\lambda_k/2)\sinh(\lambda_k/2) = 0, \tag{19}$$

implying $\cosh(\lambda_k/2) = 0$, hence $\lambda_k^2 = -(2k-1)^2\pi^2$.

Eliminating Θ and h from (1) the differential equation (7) in W follows. Consider only the case of those a, R, Q for which the associated characteristic equation $\mu^3 - Q(\mu^2 + a^2\mu) + Ra^2 = 0$, where $\mu = (\lambda^2 - a^2)$, has no multiple solutions, so the general form of the solution of (7) is

$$W(z) = \sum_{i=1}^{3}[A_i\cosh(\lambda_i z) + B_i\sinh(\lambda_i z)]. \tag{20}$$

Since imposing to this λ_k to satisfy the characteristic equation, the eigenvalue R follows, apparently the boundary conditions (2) play no role in the determination of the eigenvalue. This is not true because for the boundary condition (3) the corresponding secular equations has another splitting. The boundary conditions on W, i.e. $W = D^2W = D^4W = 0$ at $z = \pm 0.5$ can be written as

$$\begin{cases} \sum_{i=1}^{3}[A_i \cosh(\lambda_i/2) + B_i \sinh(\lambda_i/2)] = 0, \\ \sum_{i=1}^{3}[A_i\lambda_i^2 \cosh(\lambda_i/2) + B_i\lambda_i^2 \sinh(\lambda_i/2)] = 0, \\ \sum_{i=1}^{3}[A_i\lambda_i^4 \cosh(\lambda_i/2) + B_i\lambda_i^4 \sinh(\lambda_i/2)] = 0, \end{cases}$$

and lead to the secular equation $\Delta_1 \cdot \Delta_2 = 0$, where

$$\Delta_1 = \sinh(\lambda_1/2)\sinh(\lambda_2/2)\sinh(\lambda_3/2)(\mu_3 - \mu_1)(\mu_3 - \mu_2)(\mu_2 - \mu_1),$$

$$\Delta_2 = \cosh(\lambda_1/2)\cosh(\lambda_2/2)\cosh(\lambda_3/2).$$

In general, the boundary conditions-dependent equation $\Delta_1 = 0$ yields the eigenvalue leading to the neutral manifold, but as we have seen, in our case this equation has no solution. The boundary conditions-independent equation $\Delta_2 = 0$ is usually disregarded due to the fact that the hiperbolic cosine is nonvanishing. However, this is true only for real λ_k while, due to the involved physical parameters, λ_k can be purely imaginary numbers too. For instance, assume that $\lambda_1 = i|\lambda_1|(i = \sqrt{-1})$. In this case equation (19) implies $\cosh(\lambda_1/2) = 0$ whence $|\lambda_1|/2 = (2n-1)\pi/2$. Next, imposing to λ_1 to satisfy the characteristic equation, the neutral manifold follows. The equation of this manifold is very simple and permitted to easily make explicit the eigenvalue R (the Rayleigh number), similar to the one from [5].

3 Conclusions

By using three analytical methods the eigenvalue problem associated with a magnetothermoconvection equilibria problem is solved. They are based either on the characteristic equation or on the knowledge of the Fourier series expansions or of another closed forms of the eigenfunctions. In the eigenvalue equation some factors depending only on the equations and not on the boundary conditions are special. They yield the eigenvalues associated with the neutral instability for the case of free boundaries.

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