A SYMBOLIC ALGORITHM FOR THE APPROXIMATE SOLUTION OF AN INVERSE PROBLEM FOR LINER KINETIC EQUATION

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Abstract

The problem of finding a solution and the right-hand side of the kinetic equation in the case where the values of the solution are known on the boundary of a domain is considered. A new symbolic algorithm is constructed to obtain the approximated analytical solution of the problem. A computer program is presented using computer algebra system Maple.

1 Introduction

Inverse problems are problems of determining coefficients, the right-hand side, initial conditions or boundary conditions of a differential equation from some additional information about a solution of the equation. Such problems appear in many important applications of physics, geophysics, technology and medicine.

One of the characteristic features of inverse problems for differential equations is their being ill-posed in the sense of Hadamard. The general theory of ill-posed problems and their applications is developed by A. N. Tikhonov, V. K. Ivanov, M. M. Lavrent'ev and their students [7-8,14-17].

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Inverse problems for kinetic equations appear to be important both from theoretical and practical points of view. Interesting results in this field are presented in Amirov [1-4], Hamaker, Smith, Solmon and Wagner [9], Pestov and Sharafutdinov [13], Anikonov and Amirov [19], Anikonov [20]. A symbolic algorithm for computing an approximated analytic solution of three-dimensional inverse problem for the transport equation is examined by Güyer and Mirasyedioğlu [10]. Some important results devoted to numerical solving of integral geometry problems and inverse problems for kinetic equations are presented in [5- 6, 12].

In this work, a symbolic algorithm based on Galerkin method is constructed to calculate approximate analytical solution u and right-hand side λ of the kinetic equaton. A Maple program is given according to this algorithm in the last section of the paper. The physical interpretation of such problems consists in finding forces of particle interaction, scattering indicatrices, radiation sources and other physical parameters.

2 Statement of the Problem and Some Theoretical Results

The notations to be used in the paper are introduced below:

For a bounded domain G, $C^m(G)$ is the Banach space of functions that are m times continuously differentiable in G; $C^\infty(G)$ is the set of functions that belong to $C^m(G)$ for all $m \geq 0$; $C_0^\infty(G)$ is the set of finite functions in G that belong to $C^\infty(G)$; $L_2(G)$ is the space of measurable functions that are square integrable in G, $H^k(G)$ is the Sobolev space and $\overset{\circ}{H}{}^k(G)$ is the closure of $C_0^\infty(G)$ with respect to the norm of $H^k(G)$.

These standard spaces are described in detail, for example, in Lions and Magenes [11] and Mikhailov [18].

Let Ω be a domain in the Euclidean space $\mathbb{R}^{2n}$, $n \geq 1$. For the variables $(x, v) \in \Omega$, it is assumed that $x \in D$, $v \in G$, where D and G are domains in $\mathbb{R}^n$ with boundaries of class C^2. The boundary of Ω is $\partial\Omega = \overline{\Gamma}_1 \cup \overline{\Gamma}_2$, where $\Gamma_1 = \partial D \times G$, $\Gamma_2 = D \times \partial G$. We denote $H_{1,C}(\Omega)$ by the set of real-valued functions $u(x, v) \in L_2(\Omega)$, having generalized derivatives u_{x_i}, u_{v_i}, $u_{x_i v_i}$,which belong to $L_2(\Omega)$, where $i = 1, 2, ..., n$; $\overset{\circ}{H}_{1,C} = \overset{\circ}{H}_1 \cap H_{1,C}$.

In this study, the following linear kinetic equation is considered in Ω:

$$Lu \equiv \{u, H\} \equiv \sum_{i=1}^{n} \left(\frac{\partial H}{\partial v_i} \frac{\partial u}{\partial x_i} - \frac{\partial H}{\partial x_i} \frac{\partial u}{\partial v_i} \right) = \lambda(x, v), \qquad (2.1)$$

with the right-hand side λ such that

$$\left\langle \lambda, \widehat{L}\eta \right\rangle = 0, \tag{2.2}$$

for any $\eta \in H_{1,2}(\Omega)$ whose trace on $\partial\Omega$ is zero. Here

$$\widehat{L} = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i \partial v_i}, \tag{2.3}$$

$\{u, H\}$ is the Poisson bracket of u and H.

It is easy to check that the condition (2.2) holds, for example, for any function λ of the form $\lambda = g(x) + \psi(v)$, where g and ψ are continuously differentiable functions.

Equation (2.1) is extensively used in plasma physics and astrophysics. In applications, u represents the number (or the mass) of particles in the unit volume element of the phase space in the neighbourhood of the point (x, v), $\nabla_x H$ is the force acting on a particle, λ is the collisional term characterizing the variation of u caused by particle collisions.

We define $\widetilde{C}_0^3 = \left\{\varphi : \varphi \in C^3(\Omega), \varphi = 0 \text{ on } \partial\Omega\right\}$ and select a subset of $\widetilde{C}_0^3$, $\{w_1, w_2, ...\}$, which is orthonormal and everywhere dense in $L_2(\Omega)$. Let P_n be the orthogonal projector of $L_2(\Omega)$ onto M_n, where M_n is the linear span of $\{w_1, w_2, ..., w_n\}$. By $\Gamma(A)$ the set of functions u is denoted with the following properties:

i. $u \in \Gamma(A)$, $Au \in L_2(\Omega)$ in the generalized sense, where $Au = \widehat{L}Lu$;

ii. There exists a sequence $\{u_k\} \subset \widetilde{C}_0^3$ such that $u_k \xrightarrow{w} u$ in $L_2(\Omega)$ and $\langle Au_k, u_k\rangle \to \langle Au, u\rangle$ as $k \to \infty$.

The condition that $Au \in L_2(\Omega)$ in the generalized sense means that there exists a function $f \in L_2(\Omega)$ such that $\langle u, A^*\varphi\rangle = -\langle f, \varphi\rangle$ and $Au = f$ for all $\varphi \in C_0^\infty(\Omega)$, where A^* is the differential operator conjugate to A in the sense of Lagrange. It is easy to prove that

$$H_3 \cap H_1^0 \subset \Gamma'' \cap \overset{\circ}{H}_{1,C} \subset \Gamma(A) \subset L_2(\Omega),$$

where $\Gamma''(A)$ is the set of functions $u \in L_2(\Omega)$ such that $Au \in L_2(\Omega)$ in the generalized sense.

Problem 1. *Determine a pair of functions (u, λ) defined in Ω from equation (2.1), the given Hamiltonian $H(x, v) \in C^2(\Omega)$, and the trace of the solution u on the boundary $\partial\Omega$, i.e.*

$$u|_{\partial\Omega} = u_0.$$

We know the following uniqueness theorem for the inverse Problem 1, see [4] page 60.

Theorem 1. *Assume that* $H \in C^2\left(\overline{\Omega}\right)$ *and the following inequalities hold for all* $\xi \in \mathbb{R}^n$*,* $(x, v) \in \overline{\Omega}$

$$\sum_{i,j=1}^{n} \frac{\partial^2 H}{\partial v_i \partial v_j} \xi^i \xi^j \geq \alpha \left|\xi\right|^2, \quad \sum_{i,j=1}^{n} \frac{\partial^2 H}{\partial x_i \partial x_j} \xi^i \xi^j \leq 0,$$

where α *is a positive number. Then Problem 1 has at most one solution* (u, λ) *such that* $u \in \Gamma(A)$ *and* $\lambda \in L_2(\Omega)$.

It is noteworthy that Problem 1 will have infinitely many solutions if condition (2.2) is not imposed on λ, which is evident from the following example. Suppose that some function $w \in C^2\left(\overline{\Omega}\right)$ coincides with the given function u_0 on ∂D, where $u_0 = 0$ on Γ_2. Such functions exist, then the pair (w, Lw) is a solution to Problem 1.

Problem 2. *Given the equation*

$$Lu = \lambda + F,$$

where λ *satisfies (2.2) and* F *is a known function in* $H_2(\Omega)$*, find the pair of functions* (u,λ) *under the condition that*

$$u|_{\partial\Omega} = 0.$$

Problem 1 can be reduced to Problem 2, see [4] page 65. For example, if $\partial D \in C^{\left[\frac{n+1}{2}\right]+3}$, $u_0 \in H^{\frac{n+1}{2}+2}(\partial\Omega)$ and $u_0 = 0$ on Γ_2 then there exists a function $w \in H^{[n/2]+3}(\Omega)$ such that $w = 0$ on Γ_2. For any fixed "section" $\widetilde{\Gamma}_1 = \partial D \times v_0$ of the surface Γ_1 a function $w(x, v_0)$ can be constructed such that $w(x, v_0) \in H^{[n/2]+3}(D), w(x, v_0) = u_0(x, v_0)$ on $\widetilde{\Gamma}_1$ and $\|w\|_{H^{[n/2]+3}} \leq C\|u_0\|_{H^{(n+5)/2}}$ [11, 18]. Since $u_0 \in H^{(n+5)/2}$, it is obvious that $w \in H^{n/2+3}(\Omega)$ and $w = 0$ on Γ_2. Let $u_1 = u - w$, the function u_1, taken for u, satisfies the equation $Lu = \lambda - F$ with $F = Lw$ and the condition $u|_{\partial\Omega} = 0$.

The next result is true for Problem 2, see [4], page 63.

Theorem 2. *Assume* $H \in C^2\left(\overline{\Omega}\right)$ *and the following inequalities hold for all* $(x, v) \in \overline{\Omega}, \xi \in \mathbb{R}^n$*:*

$$\sum_{i,j=1}^{n} \frac{\partial^2 H}{\partial v_i \partial v_j} \xi^i \xi^j \geq \alpha_1 \left|\xi\right|^2, \quad \sum_{i,j=1}^{n} \frac{\partial^2 H}{\partial x_i \partial x_j} \xi^i \xi^j \leq -\alpha_2 \left|\xi\right|^2,$$

where α_1 *and* α_2 *are positive numbers and* $F \in H_2(\Omega)$*. Then there exists a solution* (u,λ) *of Problem 2 such that* $u \in \Gamma(A)$*,* $u \in H_1(\Omega)$*,* $\lambda \in L_2(\Omega)$*.*

3 Algorithm of Solving the Inverse Problem

An approximate solution to the problem will be sought in the following form

$$u_N = \sum_{i=1}^{N} \alpha_{N_i} w_i.$$

For computing, $n = 3$ is taken, and two specific domains D and G are chosen. Let's take the domains

$$D = \left\{ x \in \mathbb{R}^3 : |x| < 1 \right\},$$

and

$$G = \left\{ v \in \mathbb{R}^3 : |v| < 1 \right\},$$

and consider corresponding complete systems

$$\left\{ x_1^{i_1} x_2^{i_2} x_3^{i_3} \right\}_{i_1,i_2,i_3=0}^{\infty}, \left\{ v_1^{i_4} v_2^{i_5} v_3^{i_6} \right\}_{i_4,i_5,i_6=0}^{\infty},$$

in $L_2(D)$ and $L_2(G)$ respectively, where $x = (x_1, x_2, x_3)$,$v = (v_1, v_2, v_3)$.

The approximated solution can be written in the following form:

$$u_N = \sum_{i_1,i_2,i_3,i_4,i_5,i_6=1}^{N} \alpha_{N_{i_1,i_2,i_3,i_4,i_5,i_6}} w_{i_1,i_2,i_3,i_4,i_5,i_6} \eta(x) \mu(v), \tag{3.1}$$

where

$$w_{i_1,i_2,i_3,i_4,i_5,i_6} = \left\{ x_1^{i_1} x_2^{i_2} x_3^{i_3} v_1^{i_4} v_2^{i_5} v_3^{i_6} \right\}_{i_1,i_2,i_3,i_4,i_5,i_6=0}^{\infty},$$

$$\eta(x) = \left\{ \begin{array}{cc} 1 - x_1^2 - x_2^2 - x_3^2, & |x| < 1 \\ 0, & |x| \geq 1 \end{array} \right.$$

and

$$\mu(v) = \left\{ \begin{array}{cc} 1 - v_1^2 - v_2^2 - v_3^2, & |v| < 1 \\ 0, & |v| \geq 1 \end{array} \right. .$$

In expression (3.1), unknown coefficients $\alpha_{N_{i_1,i_2,i_3,i_4,i_5,i_6}}, i_1, i_2, i_3, i_4, i_5, i_6 = \overline{1, N}$ are determined from the following system of linear algebraic equations (SLAE):

$$\sum_{i_j, j=\overline{1,6}}^{N} \left(A \left(\alpha_{N_{i_1,i_2,i_3,i_4,i_5,i_6}} w_{i_1,i_2,i_3,i_4,i_5,i_6} \right) \eta(x) \mu(v), w_{i'_1,i'_2,i'_3,i'_4,i'_5,i'_6} \right)_{L_2(\Omega)} \tag{3.2}$$

$$= \left(\mathcal{F}, w_{i_1',i_2',i_3',i_4',i_5',i_6'}\right)_{L_2(\Omega)},$$

where $i_1', i_2', i_3', i_4', i_5', i_6' = \overline{1, N}$.

Algorithm 1. ($LeftSLAE$)

Left side of each equation in (3.2) is constructed.
INPUT: The order of approximation N, $i_1', i_2', i_3', i_4', i_5', i_6', \mu(x)$
OUTPUT: Left hand side of each equation in (3.2) LeftSum
Initialization
Set LeftSum=0;
$w_{i_1,i_2,i_3,i_4,i_5,i_6} = x_1^{i_1} x_2^{i_2} x_3^{i_3} v_1^{i_4} v_2^{i_5} v_3^{i_6}$
Implementation
For $i_1 = 1, ..., N$ *do For* $i_2 = 1, ..., N$ *do For* $i_3 = 1, ..., N$ *do*
For $i_4 = 1, ..., N$ *do For* $i_5 = 1, ..., N$ *do For* $i_6 = 1, ..., N$ *do*
LeftSum=LeftSum+

$$\left(A\left(\alpha_{N_{i_1,i_2,i_3,i_4,i_5,i_6}} w_{i_1,i_2,i_3,i_4,i_5,i_6}\right) \eta(x) \mu(v), w_{i_1',i_2',i_3',i_4',i_5',i_6'}\right)_{L_2(\Omega)}$$

end i_6 *end* i_5 *end* i_4 *end* i_3 *end* i_2 *end* i_1
STOP (The procedure is complete.)

Algorithm 2. (u_N, λ)

This algorithm computes the approximate analytical solution using Algorithm 1.

INPUT: The order of approximation: N, given function on the right hand side of Problem 2: $f(x, v)$ *and Hamiltonian:* $H(x, v)$
OUTPUT: Approximate solution u_N *and lambda*
Initialization
$SLAE = \{\}$
$u_N = 0$
$w_{i_1,i_2,i_3,i_4,i_5,i_6} = x_1^{i_1} x_2^{i_2} x_3^{i_3} v_1^{i_4} v_2^{i_5} v_3^{i_6}$
$F := \frac{\partial^2 f}{\partial x_1 \partial v_1} + \frac{\partial^2 f}{\partial x_2 \partial v_2} + \frac{\partial^2 f}{\partial x_3 \partial v_3}$
$\eta(x) = 1 - x_1^2 - x_2^2 - x_3^2$
$\mu(v) = 1 - v_1^2 - v_2^2 - v_3^2$
Implementation
For $i_1' = 1, ..., N$ *do For* $i_2' = 1, ..., N$ *do For* $i_3' = 1, ..., N$ *do*
For $i_4' = 1, ..., N$ *do For* $i_5' = 1, ..., N$ *do For* $i_6' = 1, ..., N$ *do*
SLAE=SLAE$\cup$

$$\left\{LeftSLAE\left(i_1', i_2', i_3', i_4', i_5', i_6', N, \eta(x), \mu(v), w_{i_1',i_2',i_3',i_4',i_5',i_6'}\right)\right\}$$

$= \left(F, w_{i_1',i_2',i_3',i_4',i_5',i_6'}\right)_{L_2(\Omega)}$
end i_6' *end* i_5' *end* i_4' *end* i_3' *end* i_2' *end* i_1'
Solve $\left(SLAE, \left\{\alpha_{N_{i_1,i_2,i_3,i_4,i_5,i_6}}\right\}\right)$
Principle Part
For $i_1 = 1, ..., N$ *do For* $i_2 = 1, ..., N$ *do For* $i_3 = 1, ..., N$ *do*
For $i_4 = 1, ..., N$ *do For* $i_5 = 1, ..., N$ *do For* $i_6 = 1, ..., N$ *do*
$u_N = u_N + \left(\alpha_{N_{i_1,i_2,i_3,i_4,i_5,i_6}} w_{i_1,i_2,i_3,i_4,i_5,i_6}\right).\eta(x)\mu(v)$
end i_6 *end* i_5 *end* i_4 *end* i_3 *end* i_2 *end* i_1
$\lambda(x,v) = \left(\frac{\partial H}{\partial v_1}\frac{\partial u_N}{\partial x_1} - \frac{\partial H}{\partial x_1}\frac{\partial u_N}{\partial v_1}\right) + \left(\frac{\partial H}{\partial v_2}\frac{\partial u_N}{\partial x_2} - \frac{\partial H}{\partial x_2}\frac{\partial u_N}{\partial v_2}\right)$
$+ \left(\frac{\partial H}{\partial v_3}\frac{\partial u_N}{\partial x_3} - \frac{\partial H}{\partial x_3}\frac{\partial u_N}{\partial v_3}\right) - f(x,v)$
End of the Algorithm 2.

Theorem 3. *Algorithm 2 computes the coefficients*

$$\left\{\alpha_{N_{i_1,i_2,i_3,i_4,i_5,i_6}}\right\}_{i_1,i_2,i_3,i_4,i_5,i_6=1}^{N}$$

in the system of linear algebraic equations (3.2) as unique under the hypotheses of Theorem 2.

Proof. It has now been proven that under the hypotheses of the Theorem 2 the system (3.2) has a unique solution $\left(\alpha_{N_{i_1,i_2,i_3,i_4,i_5,i_6}}\right)$, $i_1, i_2, i_3, i_4, i_5, i_6 = 1\,(1)\,N$ for any function $F \in H_2(\Omega)$. For this purpose, consider homogeneous version of the system:

$$\left(\sum_{i_j, j=\overline{1,6}}^{N} A\left[\alpha_{N_{i_1,i_2,i_3,i_4,i_5,i_6}} w_{i_1,i_2,i_3,i_4,i_5,i_6}\right]\eta(x)\mu(v), w_{i_1',i_2',i_3',i_4',i_5',i_6'}\right)_{L_2(\Omega)}$$

$$= 0.$$

The $(i_1, i_2, i_3, i_4, i_5, i_6)$th equation of the system is multiplied by $-2\alpha_{N_{i_1,i_2,i_3,i_4,i_5,i_6}}$ and sum from 1 to N with respect to $i_1, i_2, i_3, i_4, i_5, i_6$ we obtain

$$-2\left\langle A_N, u_N\right\rangle = 0.$$

If the following identity is considered

$$\begin{aligned}\sum_{j=1}^{n}\frac{\partial u_N}{\partial x_j}\frac{\partial}{\partial v_j}\left(Lu_N\right) &= \frac{1}{2}\sum_{i,j=1}^{n}\left(\frac{\partial^2 H}{\partial v_i \partial v_j}\frac{\partial u_N}{\partial x_i}\frac{\partial u_N}{\partial x_j}-\frac{\partial^2 H}{\partial x_i \partial x_j}\frac{\partial u_N}{\partial v_i}\frac{\partial u_N}{\partial v_j}\right)\\ &+\frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial}{\partial v_j}\left[\frac{\partial u_N}{\partial x_j}\left(\frac{\partial u_N}{\partial x_i}\frac{\partial H}{\partial v_i}-\frac{\partial u_N}{\partial v_i}\frac{\partial H}{\partial x_i}\right)\right]\\ &-\frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial}{\partial x_j}\left[\frac{\partial u_N}{\partial v_j}\left(\frac{\partial u_N}{\partial x_i}\frac{\partial H}{\partial v_i}-\frac{\partial u_N}{\partial v_i}\frac{\partial H}{\partial x_i}\right)\right]\\ &+\frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left(\frac{\partial H}{\partial v_i}\frac{\partial u_N}{\partial x_j}\frac{\partial u_N}{\partial v_j}\right)\\ &-\frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial}{\partial v_i}\left(\frac{\partial H}{\partial x_i}\frac{\partial u_N}{\partial x_j}\frac{\partial u_N}{\partial v_j}\right),\end{aligned}$$

and the geometry of the domain Ω and the condition that $u_N = 0$ on $\partial\Omega$ are taken into account, then

$$-2\left\langle Au_N, u_N\right\rangle = 2J\left(u_N\right) = 0$$

is obtained, where

$$J\left(u_N\right) \equiv \frac{1}{2}\sum_{i,j=1}^{n}\int_{\Omega}\left(\frac{\partial^2 H}{\partial v_i \partial v_j}\frac{\partial u_N}{\partial x_i}\frac{\partial u_N}{\partial x_j}-\frac{\partial^2 H}{\partial x_i \partial x_j}\frac{\partial u_N}{\partial v_i}\frac{\partial u_N}{\partial v_j}\right)d\Omega.$$

The assumptions of Theorem 2 imply $\nabla u_N = 0$. Hence $u_N = 0$ in Ω as a result of the conditions $u_N = 0$ on $\partial\Omega$ and $u \in \widetilde{C}_0^3\left(\Omega\right)$. Since the system

$$\left\{w_{i_1,i_2,i_3,i_4,i_5,i_6}\right\},\ i_1,i_2,i_3,i_4,i_5,i_6 = 1,...,N,$$

is linearly independent, we get $\alpha_{N_{i_1,i_2,i_3,i_4,i_5,i_6}} = 0, i_1,i_2,i_3,i_4,i_5,i_6 = 1,...,N$. Thus the homogeneous version of the system of linear algebraic equations (3.2) has only trivial solution and therefore the original inhomogeneous system (3.2) has a unique solution set $\left\{\alpha_{N_{i_1,i_2,i_3,i_4,i_5,i_6}}\right\}, i_1,i_2,i_3,i_4,i_5,i_6 = 1,...,N$ for any function $F \in H_2\left(\Omega\right)$. □

3.1 The Program Codes of Algorithm via MAPLE 10

Here, algorithm of solving the inverse problem for three dimensional Linear Kinetic Euation is implemented using MAPLE 10. The program computes the approximate analytic solution of the problem according to given parametres N, f, H, where N is the aproximation level, f is the right hand side of Problem 1, H is the Hamiltonian.

```
U := proc(N, f, H)
Local UN, lambda, s;
Global L2InnerProduct, operatorL, operatorLTilda, operatorA, W, a, i1,
i2,
i3, i4, i5, i6, j1, j2, j3, j4, j5, j6, SLAE, LeftHandSide, F, eta, mu;
Option Remember;
a:= array(1..N,1..N,1..N,1..N,1..N,1..N);
eta := 1 - x1^2 - x2^2 - x3^2;
mu := 1 - x1^2 - x2^2 - x3^2;
L2InnerProduct := proc(u, v)
Local k1, k2, k3;
k1 := simplify(int(u*v, x1 = - sqrt(1- x2^2 - x3^2)..sqrt(1- x2^2 - x3^2)));
k2 := simplify(int(int(k1, x2 = - sqrt(1- x1^2)..sqrt(1- x1^2)), x3 = -1..1));
k3 := simplify(int(k2, v1 = - sqrt(1- v2^2 -v3^2)..sqrt(1 - v2^2 - v3^2)));
RETURN(simplify(int(int(k3, v2 = - sqrt(1- v1^2)..sqrt(1- v1^2)), v3 =
-1..1)));
end;
operatorL := proc(U)
RETURN(diff(H,v1) * diff(U,x1) - diff(H,x1)*diff(U,v1) + diff(H,v2) *
diff(U,x2)
- diff(H,x2) * diff(U,v2) + diff(H,v3) * diff(U,x3) - diff(H,x3) * diff(U,v3));
end;
operatorLTilda := proc(U)
RETURN(diff(diff(U,v1),x1) + diff(diff(U,v2),x2) + diff(diff(U,v3),x3))
end;
operatorA := proc(U)
RETURN(operatorLTilda(operatorL(U)));
end;
F := operatorLTilda(f);
W := proc(i1,i2,i3,i4,i5,i6::nonnegint)
RETURN(x1^i1 * x2^i2 * x3^i3 * v1^i4 * v2^i5 * v3^i6);
end;
LeftHandSide := proc(j1,j2,j3,j4,j5,j6)
Local LeftSum;
LeftSum:= 0;
for i1 from 1 to N do for i2 from 1 to N do for i3 from 1 to N do
```

```
for i4 from 1 to N do for i5 from 1 to N do for i6 from 1 to N do
LeftSum := LeftSum + L2InnerProduct(operatorA(a[i1,i2,i3,i4,i5,i6] *
W(i1,i2,i3,i4,i5,i6)*eta*mu), W(j1,j2,j3,j4,j5,j6));
od; od; od; od; od; od;
RETURN(LeftSum);
end;
SLAE := proc()
Local SysEqu;
SysEqu := {};
for j1 from 1 to N do for j2 from 1 to N do for j3 from 1 to N do
for j4 from 1 to N do for j5 from 1 to N do for j6 from 1 to N do
SysEqu := {op(SysEqu), LeftHandSide(j1,j2,j3,j4,j5,j6)
= L2InnerProduct(F, W(j1,j2,j3,j4,j5,j6) * eta*mu)}
od; od; od; od; od; od;
RETURN(SysEqu);
end;
s := 0;
for i1 from 1 to N do for i2 from 1 to N do for i3 from 1 to N do
for i4 from 1 to N do for i5 from 1 to N do for i6 from 1 to N do
s := s + (a[i1,i2,i3,i4,i5,i6]*W(i1,i2,i3,i4,i5,i6))*eta*mu;
od; od; od; od; od; od;
UN := subs(solve(SLAE()), s); print('UN', UN);
lambda :=operatorL(UN)-f; print('lambda', lambda);
end;
```

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