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## Book Reviews

Clifford Henry Taubes, *Modeling Differential Equations in Biology*, Second edition. Cambridge University Press, Cambridge, 2008, xxiv+500 pp., ISBN: 978-0-521-70843-2. Price: £29.99 (US\$ 60.00).

The title precisely reflects the contents of the book, a valuable addition to the growing literature in mathematical biology from a deterministic modeling approach. This book is a suitable textbook for multiple purposes: for applied qualitative theory of differential equations and for mathematical biology, and at multiple levels: for senior undergraduate students majoring in applied mathematics, for graduate students in applied mathematics, and for students in theoretical ecology interested in mathematical modeling.

The main topics included in this second edition are the following: ordinary differential equations of a single or a pair of unknown functions, phase plane analysis, equilibrium and stability analysis, elementary linear algebra, partial derivatives, the elementary partial differential equations of advection and diffusion and their special solutions, separation of variables solutions. The author is also concerned with the stability of steady states of scalar reaction diffusion equations on bounded intervals and the traveling waves for the Fisher equation are also considered. In the last part of the book there are studied ordinary differential equations and periodic solutions (a simplified Poincaré–Bendixson theorem), fast-slow systems and chaos. Biological applications treated by the author include population dynamics (exponential growth, logistic growth, predator-prey dynamics) simple epidemic models, and epidemic waves. The included research articles greatly expand the application areas.

This book is appropriate for students with mathematical backgrounds. It poses questions of current research interest, providing a comprehensive overview of the field and a solid foundation for interdisciplinary research in the biological sciences. To cover the whole book the students will need backgrounds in linear algebra, vector calculus, difference equations, and ordinary and partial differential equations. Only methods are required, and the necessary results are collected together in appendices. This book includes many exercises as well as detailed solutions for them. All in all, it is an excellent

introduction for those beginners that are interested in the fast growing field of mathematical biology.

Vicențiu D. Rădulescu

Antonio Ambrosetti & Andrea Malchiodi, *Nonlinear Analysis and Semilinear Elliptic Problems*, Cambridge Studies in Advanced Mathematics (No. 104), Cambridge University Press, Cambridge, 2007, xii+316 pp. ISBN: 978-0-521-86320-9; 0-521-86320-1. Price: £41.00 (US\$ 82.00).

Nonlinear analysis is a quite young area in mathematical sciences, and it has grown tremendously in the last thirty years. Definitely, nonlinear analysis has become a fundamental part of the mainstream research in contemporary mathematical analysis. Many nonlinear analysis problems arise from the areas of astronomy, atmospheric sciences, fluid dynamics and elasticity, physics, chemistry, biology, control theory, image processing and economics, to name a few.

A significant number of nonlinear analysis problems originate from areas of mathematics, such as optimization theory, game theory, partial and ordinary differential equations, mathematical physics, harmonic analysis, convex analysis, and linear and nonlinear functional analysis. Indeed, this diversity of areas has generated various approaches to solving nonlinear problems, some of which are discussed in this monograph. The book emphasizes the use of variational methods in the study of differential equations. It also presents differential calculus and degree theory tools, but subordinates the use of them to certain steps of the variational method. This book collects together some recent work, mostly due to the authors (and some co-workers), concerning the qualitative analysis of semilinear elliptic equations.

The main topics included in this book are the following: Preface; Preliminaries; Part I. Topological Methods: 2. A primer on bifurcation theory; 3. Topological degree, I; 4. Topological degree, II: global properties; Part II. Variational Methods, I: 5. Critical points: extrema; 6. Constrained critical points; 7. Deformations and the Palais-Smale condition; 8. Saddle points and min-max methods; Part III. Variational Methods, II: 9. Lusternik-Schnirelman theory; 10. Critical points of even functionals on symmetric manifolds; 11. Further results on elliptic Dirichlet problems; 12. Morse theory; Part IV. Appendices: Appendix 1. Qualitative results; Appendix 2. The concentration compactness principle; Appendix 3. Bifurcation for problems on  $\mathbb{R}^n$ ; Appendix 4. Vortex rings in an ideal fluid; Appendix 5. Perturbation methods; Appendix 6. Some problems arising in differential geometry; Bibliography; Index.

This interesting book is a valuable contribution to the literature. It should be useful for a graduate course. The authors give an introduction to abstract methods in nonlinear analysis and applications to partial differential equations. They include preliminary material on linear analysis and try to give methods in enough generality to make the material interesting without trying to obtain the most general results. In my opinion the book should be strongly recommended to anyone –graduate student or researcher– who is interested in modern methods in nonlinear analysis and their applications to partial differential equations.

Overall, the book presents a unified approach, and is an excellent contribution to nonlinear analysis.

Vicențiu D. Rădulescu

D.J.H. Garling, *Inequalities: A Journey into Linear Analysis*, Cambridge University Press, Cambridge, 2007, xii+316 pp. Paperback (ISBN-13:9780521699730. Price: £24.99; US\$ 47.00). Hardback (ISBN-13:9780521876247. Price: £64.00; US\$ 122.00).

This book provides an introduction to linear analysis and a selection of inequalities that are used in linear analysis. The book is very interesting, well written, readable and should be of interest to more than just analysts. The mathematical analysts, physicists, engineers as well as many other people will find in this work a useful source of reference and inspiration.

The main topics covered by this book are the following: Introduction; 1. Measure and integral; 2. The Cauchy-Schwarz inequality; 3. The AM-GM inequality; 4. Convexity, and Jensen's inequality; 5. The  $L_p$  spaces; 6. Banach function spaces; 7. Rearrangements; 8. Maximal inequalities; 9. Complex interpolation; 10. Real interpolation; 11. The Hilbert transform, and Hilbert's inequalities; 12. Khintchine's inequality; 13. Hypercontractive and logarithmic Sobolev inequalities; 14. Hadamard's inequality; 15. Hilbert space operator inequalities; 16. Summing operators; 17. Approximation numbers and eigenvalues; 18. Grothendieck's inequality, type and cotype. The book also contains brief notes and remarks at the end of each chapter, which include suggestions for further reading and also a collection of exercises, of varied nature: some are five-finger exercises, but some establish results that are even deeper.

Overall the book contains a good introduction to the history and many current results concerning inequalities. It also contains some explicit unsolved problems and many implicit ones. The book is recommended as a source for middle-level mathematical courses. It can be used not only at mathematical department, but also by physicists, engineers, economists and other experts in

applied sciences who want to understand the main ideas of analysis in order to use them at the study of mathematical models of different type processes. The author offers a useful and fascinating manuscript which should be of interest and useful to graduate and postgraduate students, professional mathematicians, teachers as well as researchers. Most of them will be able to find all fundamental ideas in the field and simple and transparent proofs of many important inequalities and theorems closely related to these.

I recommend this book to people who want to learn more about linear analysis, in particular, on harmonic analysis, Banach space techniques and operator theory, but also on several related aspects in probability theory.

Vicențiu D. Rădulescu

Bela Bollobás, *The Art of Mathematics. Coffee Time in Memphis*, Cambridge University Press, Cambridge, 2006, New York, 2006. xvi+359 pp. ISBN: 978-0-521-69395-0; 0-521-69395-0. Price: £20.99; US\$ 39.00.

This is not a book about mathematics but about doing mathematics. The explicit aim of the author is to guide the thinking of an mentally investigating of problems by the reader. The reader has to use the presented text, the hints, the exercises and so on for developing subjective experience with and expertise in formulating hypotheses, finding proofs, checking hypotheses and many other valuable cognitive techniques. To attain this goal different problems are presented and the process of solution is documented in a very detailed and exemplary way.

This is a marvelous collection of problems and solutions from various areas of mathematics. All the problems are challenging and the whole book is most entertaining. The beauty of this book is not only the variety of problems and the every day appeal of many of them, but also that the questions and answers are provided separately. All the problems are elementary with respect to the necessary mathematical concepts and methods (essentially arithmetic, elementary algebra and elementary geometry) but not elementary with respect to problem solving. Such, the text can be used in high-schools and colleges at least with the high-achieving students.

The book is particularly appropriate for high school students looking for mathematical insight and for lay people with a secondary background and intellectual curiosity.

Vicențiu D. Rădulescu