



# Propagation of inhomogeneous plane waves in isotropic solid crystals

Livia Harabagiu and Olivian Simionescu-Panait

## Abstract

In this paper we study the impact of initial mechanical deformation and electric fields applied to linear elastic isotropic solid, on the propagation of inhomogeneous plane waves in such media. We derive the decomposition of the propagation condition for particular isotropic directional bivectors and we show that the specific coefficients are similar to the case of guided waves propagation in isotropic solids subject to a bias.

## 1 Introduction

Last decades, the problems related to electroelastic materials subject to incremental fields superposed on initial mechanical and electric fields have gained considerable extension, due to their complexity and to multiple applications. The basic equations of piezoelectric bodies subject to infinitesimal deformations and fields, superposed on initial deformation and electric fields, were given by Eringen and Maugin in the well-known monography [9].

While the concept of bivector is described in [6], the algebra of bivectors is well established in [3], [5] and [18]. Inhomogeneous plane waves arise in many areas of continuum mechanics .

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Important examples using this concept may be found in [1] for anisotropic elasticity, in [2] for electromagnetism, in [4] for wave propagation in porous materials.

In this paper we investigate the conditions of inhomogeneous plane waves propagation in isotropic solids subject to initial mechanical and electric fields. We obtain the components of the electroacoustic tensor for the class 2 and class  $m$  of the monoclinic system with coefficients similar to the case of guided wave propagation in isotropic solid.

## 2 Basic equations. Condition of propagation

We assume that the elastic dielectric is nonmagnetizable and conducts neither heat, nor electricity. We shall use the quasi-electrostatic approximation of the equations of balance. Furthermore, we assume that the elastic dielectric is linear and homogeneous, that the initial homogeneous deformations are infinitesimal and that the initial homogeneous electric field has small intensity. This problem may be found in the monographic chapter [12].

In this case *the homogeneous field equations* take the form:

$$\rho \ddot{\mathbf{u}} = \text{div} \boldsymbol{\Sigma}, \text{div} \boldsymbol{\Delta} = 0, \quad (1)$$

$$\text{rot } \mathbf{e} = 0 \Leftrightarrow \mathbf{e} = -\text{grad} \varphi$$

where  $\rho$  is the mass density,  $\mathbf{u}$  is the incremental displacement,  $\boldsymbol{\Sigma}$  is the incremental electromechanical nominal stress tensor,  $\boldsymbol{\Delta}$  is the incremental electric displacement vector,  $\mathbf{e}$  is the incremental electric field and  $\varphi$  is the incremental electric potential. All incremental fields involved into the above equations depend on the spatial variable  $\mathbf{x}$  and on time  $t$ .

We suppose the following *incremental constitutive equations*:

$$\Sigma_{kl} = \hat{\Omega}_{klmn} u_{m,n} + \hat{\Lambda}_{mkl} \varphi_{,m}, \quad (2)$$

$$\Delta_k = \hat{\Lambda}_{kmn} u_{n,m} + \hat{\epsilon}_{kl} e_l = \hat{\Lambda}_{kmn} u_{n,m} - \hat{\epsilon}_{kl} \varphi_{,l}.$$

In these equations  $\hat{\Omega}_{klmn}$  are the components of the instantaneous elasticity tensor,  $\hat{\Lambda}_{kmn}$  are the components of the instantaneous couplig tensor and  $\hat{\epsilon}_{kl}$  are the components of the instantaneous dielectric tensor. The instantaneous coefficients can be expressed in terms of the classical moduli of the material and on the initial applied fields as follows:

$$\begin{aligned} \hat{\Omega}_{klmn} &= c_{klmn} + \hat{S}_{kn} \delta_{lm} - e_{kmn} \hat{E}_l - e_{nkl} \hat{E}_m - \eta_{kn} \hat{E}_l \hat{E}_m, \\ \hat{\Lambda}_{mkl} &= e_{mkl} + \eta_{mk} \hat{E}_l, \end{aligned} \quad (3)$$

$$\mathring{c}_{kl} = \delta_{kl} + \eta_{kl}.$$

where  $c_{klmn}$  are the components of the constant elasticity tensor,  $e_{kmn}$  are the components of the constant piezoelectric tensor,  $\mathring{E}_l$  are the components of the initial applied electric field and  $\mathring{S}_{kn}$  are the components of the initial applied symmetric (Cauchy) stress tensor.

From the relations (3) we find the symmetry relations:

$$\begin{aligned} \mathring{\Omega}_{klmn} &= \mathring{\Omega}_{nmkl}, \\ \mathring{c}_{kl} &= \mathring{c}_{lk}. \end{aligned} \quad (4)$$

From the previous field and constitutive equations we obtain the following fundamental system of equations:

$$\begin{aligned} \mathring{\rho}\ddot{u}_l &= \mathring{\Omega}_{klmn}u_{m,nk} + \mathring{\Lambda}_{mkl}\varphi_{,mk}, \\ \mathring{\Lambda}_{kmn}u_{m,nk} - \mathring{e}_{kn}\varphi_{,nk} &= 0, \quad i = \overline{1,3}. \end{aligned} \quad (5)$$

We suppose that the incremental displacement is defined by the *inhomogeneous plane wave*:

$$\mathbf{u}(\mathbf{x}, t) = A e^{i(\mathbf{S} \cdot \mathbf{x} - \omega t)} \quad (6)$$

and the incremental electric potential is defined by

$$\varphi = \Phi e^{i(\mathbf{S} \cdot \mathbf{x} - \omega t)}, \quad (7)$$

where  $\omega$  defines the *frequency* of the wave, and is a real parameter.

Here  $\mathbf{A} = \mathbf{A}^+ + i\mathbf{A}^-$  is a complex vector defining the *mechanical amplitude*,  $\phi$  is the electric amplitude of the wave and  $\mathbf{S} = \mathbf{S}^+ + i\mathbf{S}^-$  is a complex vector denoting the *slowness bivector*. We suppose that this kind of wave propagates in an unbounded domain.

The previous relations represent a train of elliptically polarized plane waves. The waves travel in the direction of the vector  $\mathbf{S}^+$  with the slowness  $|\mathbf{S}^+|$  and are attenuated in the direction of the vector  $\mathbf{S}^-$ . For any fixed position vector  $x$ , the displacement vector  $u^+$  describes an ellipse similar to the one found in [7].

A solution of form (6) defines an "*inhomogeneous plane wave*" (IPW) if the vector  $\mathbf{S}^-$  is not parallel to the vector  $\mathbf{S}^+$ . The *phase speed* is given by  $V = |\mathbf{S}^+|^{-1}$ , while  $|\mathbf{S}^-|$  defines the *attenuation coefficient*. If  $\mathbf{S}^-$  is parallel to  $\mathbf{S}^+$  we have an *attenuated homogeneous plane wave*.

In order to solve the problem of inhomogeneous plane wave propagation in the described material, we use the *directional ellipse method*, due to Hayes (see [7]). The slowness bivector  $\mathbf{S}$  may be written as  $\mathbf{S} = N\mathbf{C}$ , where the directional bivector  $\mathbf{C}$  has form  $\mathbf{C} = q\mathbf{m} + i\mathbf{n}$ , with  $\mathbf{m} \cdot \mathbf{n} = 0$ ,  $|\mathbf{m}| = |\mathbf{n}| = 1$ ,  $|q| \geq 1$ .

$N$  is called the complex scalar slowness. Because the directional bivector  $\mathbf{C}$  is prescribed, the main unknown of the inhomogeneous plane wave propagation problem is the complex scalar slowness  $N$ .

We have:

$$\begin{aligned}
 \dot{u} &= -i\omega u, & (8) \\
 \ddot{u} &= -\omega^2 u, \\
 u_{,i} &= i\omega N C_i u, \\
 u_{,ij} &= -\omega^2 N^2 C_i C_j u, \\
 u_{l,ij} &= -\omega^2 N^2 C_i C_j u_l, \\
 \varphi_{,i} &= -\omega N C_i \varphi, \\
 \varphi_{,ij} &= -\omega^2 N^2 C_i C_j \varphi, \quad i, j, l = \overline{1,3}.
 \end{aligned}$$

Inserting (8) into (5) gives:

$$\begin{aligned}
 \dot{\rho}\omega^2 u_l &= -\omega^2 \mathring{\Omega}_{klmn} N^2 C_n C_k u_m - \mathring{\Lambda}_{mkl} \omega^2 N^2 C_m C_k \varphi, & (9) \\
 \mathring{\Lambda}_{kmn} \omega^2 N^2 C_m C_k u_n + \mathring{\varepsilon}_{kn} \omega^2 N^2 C_n C_k \varphi &= 0.
 \end{aligned}$$

From which we deduce:

$$\begin{aligned}
 \mathring{\Omega}_{klmn} N^2 C_n C_k u_m + \mathring{\Lambda}_{mkl} N^2 C_m C_k \varphi - \dot{\rho} u_l &= 0, & (10) \\
 \mathring{\Lambda}_{kmn} N^2 C_m C_k u_n - \mathring{\varepsilon}_{kn} N^2 C_n C_k \varphi &= 0.
 \end{aligned}$$

Thus:

$$\begin{aligned}
 \mathring{\Omega}_{klmn} C_n C_k u_m + \mathring{\Lambda}_{mkl} C_m C_k \varphi - \dot{\rho} \frac{u_l}{N^2} &= 0, & (11) \\
 \mathring{\Lambda}_{kmn} C_m C_k u_n - \mathring{\varepsilon}_{kn} C_n C_k \varphi &= 0.
 \end{aligned}$$

Taking  $V = \frac{1}{N}$  and  $\varphi = u_4$  we obtain:

$$\mathring{\Omega}_{klmn} C_n C_k u_m + \mathring{\Lambda}_{mkl} C_m C_k u_4 - \dot{\rho} v^2 u_l = 0, \quad (12)$$

$$\mathring{\Lambda}_{kmn}C_mC_ku_n - \mathring{\varepsilon}_{kn}C_nC_ku_4 = 0,$$

which gives:

$$\begin{pmatrix} \mathring{\Gamma}_{11} - \mathring{\rho}V^2 & \mathring{\Gamma}_{12} & \mathring{\Gamma}_{13} & \mathring{\gamma}_1 \\ \mathring{\Gamma}_{21} & \mathring{\Gamma}_{22} - \mathring{\rho}V^2 & \mathring{\Gamma}_{23} & \mathring{\gamma}_2 \\ \mathring{\Gamma}_{31} & \mathring{\Gamma}_{32} & \mathring{\Gamma}_{33} - \mathring{\rho}V^2 & \mathring{\gamma}_3 \\ \mathring{\gamma}_1 & \mathring{\gamma}_2 & \mathring{\gamma}_3 & -\mathring{\varepsilon} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = 0 \quad (13)$$

where

$$\mathring{\Gamma}_{lm} = \mathring{\Omega}_{klmn}C_nC_k = c_{klmn} + \mathring{S}_{kn}\delta_{lm} - e_{kmn}\mathring{E}_l - e_{nkl}\mathring{E}_m - \eta_{kn}\mathring{E}_l\mathring{E}_mC_kC_n, \quad (14)$$

$$\mathring{\gamma}_l = \mathring{\Lambda}_{mkl}C_mC_k = (e_{mkl} + \eta_{mk}\mathring{E}_l)C_mC_k, \quad (15)$$

$$\mathring{\varepsilon} = \mathring{\varepsilon}_{kn}C_kC_n = (\delta_{kn} + \eta_{kn})C_kC_n. \quad (16)$$

System (13) represents *the propagation condition of the inhomogeneous plane waves* inside the previous materials and is equivalent to:

$$\begin{pmatrix} \mathring{Q}_{lm} & \mathring{Q}_{l4} \\ \mathring{Q}_{4m} & \mathring{Q}_{44} \end{pmatrix} \begin{pmatrix} u_m \\ u_4 \end{pmatrix} = 0, \quad m = \overline{1,3}, \quad (17)$$

where  $Q$  is *the electroacoustic tensor* and has the following components:

$$\mathring{Q}_{lm} = N^2\mathring{\Omega}_{klmn}C_nC_k - \mathring{\rho}\delta_{lm}, \quad (18)$$

$$\mathring{Q}_{l4} = N^2\mathring{\Lambda}_{mkl}C_mC_k,$$

$$\mathring{Q}_{4m} = N^2\mathring{\Lambda}_{klm}C_lC_k,$$

$$\mathring{Q}_{44} = -N^2\mathring{\varepsilon}_{kn}C_nC_k.$$

Note that the tensor  $\mathbf{Q}$  is symmetric for the general anisotropy.

### 3 .Inhomogeneous plane waves in isotropic solids subject to initial electro-mechanical fields

In the particular case of an isotropic material, the elasticity tensor contains two independent components. Using Voight's convention we have:

$$\begin{pmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}, \quad (19)$$

with  $c_{11} = \lambda + 2\mu$ ,  $c_{12} = \lambda$  and  $c_{66} = (c_{11} - c_{12})/2 = \mu$ . Here  $\lambda$  and  $\mu$  are Lamé's coefficients.

The dielectric tensor has only one component, hence

$$\eta = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & \eta \end{pmatrix}, \quad (20)$$

and  $\varepsilon = 1 + \eta$ .

From (14) now gives:

$$\mathring{\Gamma}_{11} = (\mathring{c}_{11} + \mathring{S}_{11})C_1^2 + (\mathring{c}_{66} + \mathring{S}_{22})C_2^2 + (\mathring{c}_{66} + \mathring{S}_{33})C_3^2 - \eta(C_1^2 + C_2^2 + C_3^2)\mathring{E}_1^2 + (21)$$

$$+ 2\mathring{S}_{12}C_1C_2 + 2\mathring{S}_{13}C_1C_3 + 2\mathring{S}_{23}C_2C_3,$$

$$\mathring{\Gamma}_{12} = \mathring{\Gamma}_{21} = -\eta(C_1^2 + C_2^2 + C_3^2)\mathring{E}_1\mathring{E}_2 + (\mathring{c}_{12} + \mathring{c}_{66})C_1C_2$$

$$\mathring{\Gamma}_{13} = \mathring{\Gamma}_{31} = -\eta(C_1^2 + C_2^2 + C_3^2)\mathring{E}_1\mathring{E}_3 + (\mathring{c}_{12} + \mathring{c}_{66})C_1C_3,$$

$$\mathring{\Gamma}_{22} = (\mathring{c}_{66} + \mathring{S}_{11})C_1^2 + (\mathring{c}_{11} + \mathring{S}_{22})C_2^2 + (\mathring{c}_{66} + \mathring{S}_{33})C_3^2 - \eta(C_1^2 + C_2^2 + C_3^2)\mathring{E}_2^2 +$$

$$+ 2\mathring{S}_{12}C_1C_2 + 2\mathring{S}_{13}C_1C_3 + 2\mathring{S}_{23}C_2C_3,$$

$$\mathring{\Gamma}_{23} = \mathring{\Gamma}_{32} = -\eta(C_1^2 + C_2^2 + C_3^2)\mathring{E}_2\mathring{E}_3 + (\mathring{c}_{12} + \mathring{c}_{66})C_2C_3,$$

$$\mathring{\Gamma}_{33} = (\mathring{c}_{66} + \mathring{S}_{11})C_1^2 + (\mathring{c}_{66} + \mathring{S}_{22})C_2^2 + (\mathring{c}_{11} + \mathring{S}_{33})C_3^2 - \eta(c_1^2 + c_2^2 + c_3^2)\mathring{E}_3^2 +$$

$$+2\mathring{S}_{12}C_1C_2 + 2\mathring{S}_{13}C_1C_3 + 2\mathring{S}_{23}C_2C_3.$$

From (15) we also obtain:

$$\mathring{\gamma}_1 = (1 + \eta)(C_1^2 + C_2^2 + C_3^2)\mathring{E}_1, \quad (22)$$

$$\mathring{\gamma}_2 = (1 + \eta)(C_1^2 + C_2^2 + C_3^2)\mathring{E}_2,$$

$$\mathring{\gamma}_3 = (1 + \eta)(C_1^2 + C_2^2 + C_3^2)\mathring{E}_3.$$

and from (16) we obtain:

$$\mathring{\varepsilon} = (1 + \eta)(C_1^2 + C_2^2 + C_3^2). \quad (23)$$

### 3.1 .Direct dyad axis

In this case, we suppose that  $x_3$  is a direct dyad axis (this means the plane  $0x_1x_2$  is normal to a direct axis of order two). We have  $A_2 \parallel x_3$  and we are in the class 2 of the monoclinic system.

If we consider the particular case of *isotropic directional bivectors*, we may choose  $\mathbf{C} = (1, i, 0)$ . In this case, the inhomogeneous wave is circularly polarized in a plane normal to the dyad axis  $x_3$ .

If  $\mathring{E}_1 = \mathring{E}_2 = 0$ , we obtain  $\mathring{\Gamma}_{13} = \mathring{\Gamma}_{23} = 0$  si  $\mathring{\gamma}_1 = \mathring{\gamma}_2 = \mathring{\gamma}_3 = \mathring{\varepsilon} = 0$ . The system (17) reduces to two independent subsystems:

a) The first subsystem:

$$\begin{pmatrix} \mathring{\Gamma}_{11} - \mathring{\rho}V^2 & \mathring{\Gamma}_{12} \\ \mathring{\Gamma}_{12} & \mathring{\Gamma}_{22} - \mathring{\rho}V^2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0, \quad (24)$$

where

$$\mathring{\Gamma}_{11} = (\mathring{c}_{11} + \mathring{S}_{11}) - (\mathring{c}_{66} + \mathring{S}_{22}) + 2\mathring{S}_{12}i,$$

$$\mathring{\Gamma}_{12} = \mathring{\Gamma}_{21} = (\mathring{c}_{12} + \mathring{c}_{66})i,$$

$$\mathring{\Gamma}_{22} = (\mathring{c}_{66} + \mathring{S}_{11}) - (\mathring{c}_{11} + \mathring{S}_{22}) + 2\mathring{S}_{12}i.$$

The solution of this subsystem corresponds to  $\mathring{P}_2$  guided wave. This problem defines a non-piezoelectric guided wave, polarized in the plane  $x_1x_2$ .  $\mathring{P}_2$  depends on the initial stress field, only.

b) The solution of this subsystem is a single equation, as follows:

$$(\mathring{\Gamma}_{33} - \mathring{\rho}V^2)u_3 = 0, \quad (25)$$

where  $\mathring{\Gamma}_{33} = (\mathring{S}_{11} - \mathring{S}_{22}) + 2i\mathring{S}_{12}$ .

The solution of this subsystem is a transverse-horizontal wave, with polarization after the axis  $x_3$ . This wave is piezoelectric, depends on the initial mechanical fields, and is denoted by  $\overline{TH}$ .

### 3.2 Inverse dyad axis (mirror plane)

We now suppose the plane  $x_1x_2$  is normal to an inverse dyad axis ( $x_3$  in our case), which is equivalent to the fact that the plane  $x_1x_2$  is parallel to a mirror plane  $M$ . It follows that the material belongs to the class  $m$  of the *monoclinic system* ( $M \perp x_3$ ). In this case, the electroacoustic tensor  $Q$  is symmetric with complex components.

If we consider the particular case of isotropic directional bivectors, we may choose  $\mathbf{C} = (1, i, 0)$ . In this case, the inhomogeneous wave is circularly polarized in a normal plane to the inverse dyad axis  $x_3$ .

Moreover, if  $\mathring{E}_3 = 0$  we obtain  $\mathring{\Gamma}_{13} = \mathring{\Gamma}_{23} = 0$  and  $\mathring{\gamma}_1 = \mathring{\gamma}_2 = \mathring{\gamma}_3 = \mathring{\varepsilon} = 0$ . In this case, the system (17) reduces to two independent subsystems:

a) The first subsystem:

$$\begin{pmatrix} \mathring{\Gamma}_{11} - \mathring{\rho}V^2 & \mathring{\Gamma}_{12} \\ \mathring{\Gamma}_{12} & \mathring{\Gamma}_{22} - \mathring{\rho}V^2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0, \quad (26)$$

where

$$\mathring{\Gamma}_{11} = (\mathring{c}_{11} + \mathring{S}_{11}) - (\mathring{c}_{66} + \mathring{S}_{22}) + 2\mathring{S}_{12}i,$$

$$\mathring{\Gamma}_{12} = \mathring{\Gamma}_{21} = (\mathring{c}_{12} + \mathring{c}_{66})i,$$

$$\mathring{\Gamma}_{22} = (\mathring{c}_{66} + \mathring{S}_{11}) - (\mathring{c}_{11} + \mathring{S}_{22}) + 2\mathring{S}_{12}i.$$

A solution of this system is an inhomogeneous plane wave, polarized into the plane  $x_1x_2$  and depending on the initial stress fields. It also corresponds to  $\mathring{P}_2$  from the problem of guided wave propagation.

b) The second subsystem reduces to a single equation, as follows:

$$(\mathring{\Gamma}_{33} - \mathring{\rho}V^2)u_3 = 0, \quad (27)$$

where  $\mathring{\Gamma}_{33} = (\mathring{S}_{11} - \mathring{S}_{22}) + 2i\mathring{S}_{12}$ .

Its root is linked to a transverse-horizontal wave, with polarization after the axis  $x_3$ , non-piezoelectric and influenced by the initial stress field, only. It corresponds to  $\overline{TH}$  from the problem of guided wave propagation.



## 4 Conclusions

In this paper, we obtained the condition of inhomogeneous plane wave propagation in isotropic solid crystals subject to initial electromechanical fields. For particular isotropic directional bivectors we derive the decomposition of the propagation condition, and we show that the specific coefficients are similar to the case of guided waves propagation in isotropic solid crystals subject to a bias.

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Livia Harabagiu  
Department of Mathematics, University of Bucharest,  
Bucharest, Romania  
harabagiu\_livia@yahoo.com