



Distributive and Dual Distributive Elements in Hyperlattices

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Abstract

In this paper we introduce and study distributive elements, dual distributive elements in hyperlattices, and prove that these elements forms \wedge -semi lattice and \vee -semi hyperlattice, respectively. We use the properties of prime ideals and prime filters in hyperlattices. We will proceed to introduce the notion of dual distributive hyperlattices, I -filters and filters generated by dual distributive elements. Finally, the relationship between good homomorphisms and distributive (resp. dual distributive) elements in hyperlattices are investigated.

1 Introduction

Hyperstructure theory was first initiated by Marty [15] in 1934 when he defined hypergroups and started to analyze their properties. Since there are extensive applications in many branches of mathematics and applied sciences, the theory of algebraic hyperstructures (or hypersystems) has nowadays become a well-established branch in algebraic theory and has been developed in various fields, for more details see [1, 6, 7, 11, 14, 19, 18] and books e.g.[6, 23]. Some applications can be found also in e.g. in [4, 5, 8, 10, 16, 17]. The theory of hyperlattices was introduced by Konstantinidou in 1977 [13]. Barghi considered the prime ideal theorem for distributive hyperlattices [20]. Koguel,

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Nkuimi, and Lele studied ideals and filters in hyperlattices [12]. Rasouli and Davvaz defined fundamental relation on a hyperlattice and obtained a lattice from a hyperlattice. Moreover, they defined a topology on the set of prime ideals of a distributive hyperlattice [21, 22]. Ameri, Amiri-Bideshki, Saeid, Hoskova-Mayerova studied prime filters of hyperlattices in [2].

In this paper, we continue and deepen the ideas from [2], in particular we introduce distributive (dual distributive) elements, dual distributive hyperlattices, and I -filter in dual distributive hyperlattices. We give examples of the distributive elements and dual distributive elements in hyperlattices and we show that these concepts are independent. Also, we study \wedge -homomorphisms and \vee -homomorphisms in hyperlattices and consider relationship between (\vee) or (\wedge) homomorphisms and distributive (dual distributive) elements in hyperlattices. Finally, we define good homomorphisms and we obtain some results of them.

2 Preliminaries

In this section we give some results of hyperstructures and mainly hyperlattices that we need to develop our paper.

Definition 2.1. [15] Let H be a nonempty set and $P^*(H)$ denotes the set of all nonempty subsets of H . Maps $f: H \times H \rightarrow P^*(H)$, are called *hyperoperations*.

Definition 2.2. [13] Let L be a nonempty set, \wedge - be a binary operation, and \vee - be a hyperoperation on L . L is called a *hyperlattice* if for all $a, b, c \in L$ the following conditions hold:

$$\begin{aligned} & [(i)] a \in a \vee a, \text{ and } a \wedge a = a; a \vee b = b \vee a, \text{ and } a \wedge b = b \wedge a; a \in \\ & [a \wedge (a \vee b)] \cap [a \vee (a \wedge b)]; a \vee (b \vee c) = (a \vee b) \vee c, \text{ and } a \wedge (b \wedge c) = (a \wedge b) \wedge c; \\ & a \in a \vee b \implies a \wedge b = b. \end{aligned}$$

Let $A, B \subseteq L$. Then define:

$$A \vee B = \bigcup \{a \vee b \mid a \in A, b \in B\};$$

$$A \wedge B = \{a \wedge b \mid a \in A, b \in B\}.$$

Definition 2.3. [12] Let L be a hyperlattice. L is called *bounded* if there exist $0, 1 \in L$ such that for all $a \in L, 0 \leq a \leq 1$. We say that 0 is the *least element* and 1 is the *greatest element* of L .

Definition 2.4. [20] Let L be a hyperlattice and I be a nonempty subset of L . I is called an *ideal* if the following conditions hold:

$$[(i)] \text{ If } a, b \in I, \text{ then } a \vee b \subseteq I; \text{ If } a \in I, b \leq a, \text{ and } b \in L, \text{ then } b \in I.$$

Ideal I is called a *prime ideal* if $a \wedge b \in I$ then $a \in I$ or $b \in I$ for all $a, b \in L$.

Definition 2.5. [20] Let L be a hyperlattice and F be a nonempty subset of L . F is called a *filter* if the following conditions hold:

[(i)] If $a, b \in F$, then $a \wedge b \in F$; If $a \in F$, $a \leq b$, and $b \in L$, then $b \in F$.

Filter F is called *prime filter* if $(a \vee b) \in F \neq \emptyset$, then $a \in F$ or $b \in F$ for all $a, b \in L$. L is *distributive* if for all $a, b, c \in L$: $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$.

An example of an distributive hyperlattice can be found e.g. at [2].

Theorem 2.6. [12] Let L be a distributive hyperlattice. Then $0 \vee 0 = 0$.

Definition 2.7. [3] Let L be a nonempty set. Then L is called \wedge -*semilattice* if for all $a, b, c \in L$ the following conditions hold:

[(i)] $a \wedge a = a$; $a \wedge b = b \wedge a$; $a \wedge (b \wedge c) = (a \wedge b) \wedge c$.

3 Ideals and filters in hyperlattices

In this section, some properties of hyperlattices are mentioned and the relationship between prime ideals and prime filters in hyperlattices is discussed. Also the definitions of I -filter and the filter generated by $a \in L$ are recalled. More details can be found in [2].

In the sequel, L denotes a bounded hyperlattice.

In [2] was defined the order \leq on L by

$$a \leq b \iff b \in a \vee b \iff a \wedge b = a. \quad (3.1)$$

The binary relation \leq is reflexive, antisymmetric, and transitive, thus (L, \leq) is a poset and for all $a, b \in L$ the following conditions hold:

[(i)] $a \in a \vee 0$; $1 \in a \vee 1$; If $a, b \neq 0$ and $a \wedge b = 0$, then $a, b \notin a \vee b$.

Theorem 3.1. [2] If P is a prime ideal of L , then $L \setminus P$ is a prime filter of L .

Also holds: If P is a prime ideal of L , then $L \setminus P$ is a \wedge -semilattice. Which means, that the converse of Theorem 3.1 holds too. Now, we recall the following theorem:

Theorem 3.2. [2] Let F be a prime filter of L . Then $L \setminus F$ is a prime ideal of L .

Definition 3.3. [2] Let L be a non-empty set and \vee - be a hyperoperation on L . Then L is called a \vee -*semi-hyperlattice* if for all $a, b, c \in L$ the following conditions hold:

[(i)] $a \in a \vee a$; $a \vee b = b \vee a$; $a \vee (b \vee c) = (a \vee b) \vee c$.

In this part we recall for the readers convenient the definition of I -filters and filters generated by an element a in a type of hyperlattices that are called *dual distributive*.

Definition 3.4. [2] A hyperlattice L is called *quasi dual distributive* if for all $a, b, c \in L$.

$$a \vee (b \wedge c) \subseteq (a \vee b) \wedge (a \vee c);$$

L is called *weak dual distributive* if: $a \vee (b \wedge c) \cap [(a \vee b) \wedge (a \vee c)] \neq \emptyset$;

L is called *dual distributive* if: $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$.

Clearly a quasi dual distributive hyperlattice is weak dual distributive and a dual distributive hyperlattice is quasi distributive and weak dual distributive.

The hyperlattices such that are distributive and dual distributive, are called *strongly distributive hyperlattices*.

Lemma 3.5. [2] Let L be a dual distributive hyperlattice and $a \in L$. We define $F_a = \{x \in L : x \neq 0, 1 \in x \vee a\}$, then F_a is a filter of L .

Theorem 3.6. [2] Let L be a dual distributive hyperlattice and I be a nonempty subset of L . Define $F_I = \{x \in L : x \neq 0, 1 \in x \vee a, \forall a \in I\}$. Then

[(i)]

1. F_I is a filter of L .
2. $F_I = \bigcap_{a \in I} F_a$.
3. $F_a \cap F_b = F_{a \wedge b}$.
4. If $A \subseteq B$, then $F_B \subseteq F_A$.

F_I is called I -filter generated by I .

4 Distributive and Dual Distributive Elements

In this section, we introduce distributive and dual distributive elements in hyperlattices. Also, we study Homomorphisms and good homomorphisms in hyperlattices. Then we study the relationship between dual distributive elements and homomorphisms. Finally, we give some properties of good homomorphisms.

Assume L, L_1 and L_2 are bounded hyperlattices.

Definition 4.1. Let $a \in L$. Then:

[(i)] **The element a is called distributive, if $a \wedge (x \vee y) = (a \wedge x) \vee (a \wedge y)$, The element a is called dual distributive, if $a \vee (x \wedge y) = (a \vee x) \wedge (a \vee y)$, for all $x, y \in L$.**

Example 4.2. Let L be the hyperlattice in Example 1 [2]. Let $L = \{0, a, 1\}$, and \wedge and \vee are given with Table 1. Then the element a is distributive

\wedge	0	a	1	\vee	0	a	1
0	0	0	0	0	$\{0, a, 1\}$	$\{a, 1\}$	$\{1\}$
a	0	a	a	a	$\{a, 1\}$	$\{a\}$	$\{1\}$
1	0	a	1	1	$\{1\}$	$\{1\}$	$\{1\}$
(a)				(b)			

Table 1

element, but the element 0 is not distributive, since $0 \wedge (a \vee 1) = \{0\}$ and $(0 \wedge a) \vee (0 \wedge 1) = L$.

Corollary 4.3. *The following conditions hold.*

[(i)] L is a distributive hyperlattice if and only if for all $a \in L$, a is a distributive element. L is a dual distributive hyperlattice if and only if for all $a \in L$, a is a dual distributive element.

Let $A \subseteq L$. We say that A is a distributive subset, if and only if for all $a \in A$, a is a distributive element. Similarly, a dual distributive subset of L is defined.

Theorem 4.4. *Let $a, b \in L$. Then the following conditions hold:*

[(i)] If a and b are distributive elements, then $a \wedge b$ so is. If a and b are dual distributive elements, then the subset $a \vee b$ is a dual distributive subset.

2. Proof. Suppose $x, y \in L$. (i) We have:

$$\begin{aligned} (a \wedge b) \wedge (x \vee y) &= a \wedge b \wedge (x \vee y) = a \wedge [(b \wedge x) \vee (b \wedge y)] \\ &= [a \wedge (b \wedge x)] \vee [a \wedge (b \wedge y)] = [(a \wedge b) \wedge x] \vee [(a \wedge b) \wedge y]. \end{aligned}$$

(ii) We have

$$\begin{aligned} (a \vee b) \vee (x \wedge y) &= a \vee b \vee (x \wedge y) = a \vee [(b \vee x) \wedge (b \vee y)] \\ &= [a \vee (b \vee x)] \wedge [a \vee (b \vee y)] = [(a \vee b) \vee x] \wedge [(a \vee b) \vee y]. \quad \square \end{aligned}$$

Corollary 4.5. **[(i)]**

1. Let $D(L)$ denotes the set of all distributive elements of L . Then $D(L)$ is a \wedge - semi lattice.

**2. Let $DD(L)$ denote the set of all dual distributive elements of L .
Then $DD(L)$ is a \vee -semi-hyperlattice.**

Theorem 4.6. *Let a be a dual distributive element of L . We define $F(a) = \{x \in L : x \in x \vee a\}$, then $F(a)$ is the least filter of L which contains the element a .*

Proof. Suppose $x, y \in F(a)$, We show that $x \wedge y \in F(a)$. We have: $x \in x \vee a$ and $y \in y \vee a$ and the element a is dual distributive. Hence $x \wedge y \in (x \vee a) \wedge (y \vee a) = a \vee (x \wedge y)$, so $x \wedge y \in (x \wedge y) \vee a$. Thus $(x \wedge y) \in F(a)$. Now assume $b \leq x$ and $b \in F(a)$, we show that $x \in F(a)$. We have $b \in b \vee a$, and by Definition 3.1, $a \leq b$. Now, we have $a \leq b$, and $b \leq x$. The binary relation \leq is reflexive, antisymmetric, and transitive, thus (L, \leq) is a poset, therefore $a \leq x$ that is implied $x \in x \vee a$, so $x \in F(a)$. So $F(a)$ is a filter of L . Now, we show that $F(a)$ is the least filter of L which contains the element a . Assume F is a filter of L such that $a \in F$. We show that $F(a) \subseteq F$. Let $x \in F(a)$. So $x \in x \vee a$, by Definition 3.1, $a \leq x$. Since F is a filter and $a \in F$, so $x \in F$. Therefore $F(a) \subseteq F$. \square

Corollary 4.7. $F(a) = \{x \in L : a \leq x\} = \bigcap \{F \mid F \text{ is a filter and } a \in F\}$.

Proposition 4.8. [(i)]

1. $F(0) = L$;
2. $F(1) = \{1\}$;
3. If $a \leq b$ and a, b are two dual distributive elements, then $F(b) \subseteq F(a)$.

Corollary 4.9. *If L is a dual distributive hyperlattice, then $F(a), F(b) \subseteq F(a \wedge b)$, for all $a, b \in L$.*

Let a be a dual distributive element of hyperlattice L and $F(a)$ denote the least filter generated by a . We give a corresponding between distributive elements and \wedge -homomorphisms. Also, we assume $P(L)$ is the power set of hyperlattice L . We know that $(P(L), \cup, \cap)$ is a lattice and every lattice is a hyperlattice.

Definition 4.10. Let L and K be hyperlattices and $f : L \rightarrow K$ is mapping. Then for all $x, y \in L$

[(i)] f is called \wedge -homomorphism if:

$$f(x \wedge y) = f(x) \wedge f(y),$$

f is called \vee - homomorphism if

$$f(x \vee y) \subseteq f(x) \vee f(y),$$

f is called homomorphism if f is a \wedge - homomorphism and a \vee - homomorphism. f is called good homomorphism if the following statements hold;

$$f(x \wedge y) = f(x) \wedge f(y);$$

and

$$f(x \vee y) = f(x) \vee f(y).$$

A map f is called isomorphism if f is good, one to one, and onto homomorphism.

Example 4.11. Let L be the hyperlattice in the Example 4.2 and $f : L \rightarrow L$ is defined by: $f(0) = 0$, $f(a) = a$, and $f(1) = a$. Then f is a homomorphism, but it is not good. Since $f(0 \vee a) = \{a\}$, and $f(0) \vee f(a) = \{a, 1\}$, so $f(0 \vee a) \subset f(0) \vee f(a)$.

Lemma 4.12. Let $a \in L$. Define $f_a : L \rightarrow P(L)$ by: $f_a(x) = x \vee a$, for all $x \in L$. Then f_a is a \wedge - homomorphism, if and only if a is a dual distributive element.

- 2.** *Proof.* Let f_a be a \wedge - homomorphism and $x, y \in L$. So $f_a(x \wedge y) = f_a(x) \wedge f_a(y)$, therefore $a \vee (x \wedge y) = (a \vee x) \wedge (a \vee y)$. So by Definition 4.1 (1), the element a is dual distributive.

Conversely, suppose a is a dual distributive element. So $a \vee (x \wedge y) = (a \vee x) \wedge (a \vee y)$, and it means that $f_a(x \wedge y) = f_a(x) \wedge f_a(y)$. Therefore, f_a is a \wedge - homomorphism. \square

Theorem 4.13. The following conditions for hyperlattice L are equivalent.

[(i)] For all $a \in L$, f_a is a \wedge - homomorphism, L is a dual distributive hyperlattice.

- 2.** *Proof.* By Lemma 4.12 and Corollary 4.3, (2) proof is obvious. \square

Lemma 4.14. Let $a \in L$. Then the map $g_a : L \rightarrow P(L)$ that is defined by $g_a(x) = \{a \wedge x\}$ for all $x \in L$, is a good homomorphism, if and only if a is a distributive element.

Proof. Suppose g_a is a good homomorphism and $x, y \in L$. So $g_a(x \vee y) = g_a(x) \vee g_a(y)$, therefore $a \wedge (x \vee y) = (a \wedge x) \vee (a \wedge y)$ and this means that a is a distributive element.

Conversely, suppose the element a is distributive. So $a \wedge (x \vee y) = (a \wedge x) \vee (a \wedge y)$, and it implies that $g_a(x \vee y) = g_a(x) \vee g_a(y)$. Since $a \wedge a = a$, so:

$$g_a(x \wedge y) = a \wedge (x \wedge y) = (a \wedge a) \wedge (x \wedge y) = (a \wedge x) \wedge (a \wedge y) = g_a(x) \wedge g_a(y).$$

So g_a is a good homomorphism. \square

Theorem 4.15. *The following conditions are equivalent.*

[(i)] For all $a \in L$, g_a that is defined in Lemma 4.14 is a good homomorphism. L is a distributive hyperlattice.

2. Proof. The proof is obvious by Lemma 4.14 and Corollary 4.3 . \square

Corollary 4.16. *Let a be an element of L . Then the following conditions on the element a are equivalent:*

[(i)] a is a distributive element. The mapping $f : L \rightarrow P(F(a))$ that is defined by $f(x) = \{a \wedge x\}$, for all $x \in L$, is a good homomorphism.

For a good homomorphism f , define $Ker(f) = \{x | f(x) = 1\}$. $Ker(f)$ is called kernel of f .

Example 4.17. [(i)]

1. If $f : L \rightarrow L$ is defined by $f(x) = 1$ for all $x \in L$, then f is a good homomorphism and $Ker(f) = L$.
2. Let L be the hyperlattice in [12], i.e.: Let $L = \{0, a, b, 1\}$ and \wedge and \vee are given with Table 2

\wedge	0	a	b	1	\vee	0	a	b	1
0	0	0	0	0	0	$\{0\}$	$\{a\}$	$\{b\}$	$\{1\}$
a	0	a	0	a	a	$\{a\}$	$\{0, a\}$	$\{1\}$	$\{b, 1\}$
b	0	0	b	b	b	$\{b\}$	$\{1\}$	$\{0, b\}$	$\{a, 1\}$
1	0	a	b	1	1	$\{1\}$	$\{b, 1\}$	$\{a, 1\}$	L
(a)					(b)				

Table 2

Then $(L, \vee, \wedge, 0, 1)$ is a distributive hyperlattice. Let moreover $f : L \rightarrow L$ be defined by:

$$f(a) = b, f(b) = a, f(0) = 0, f(1) = 1,$$

then f is an isomorphism and $Ker(f) = \{1\}$.

Proposition 4.18. *Let $f : L_1 \longrightarrow L_2$ be a good homomorphism. Then the following conditions hold:*

[(i)] **If $\text{Ker}(f) \neq \emptyset$, then $\text{Ker}(f)$ is a filter of L_1 . If $x \leq y$, then $f(x) \leq f(y)$. If f is an epimorphism, then $\text{Ker}(f) \neq \emptyset$ also, $f(1) = 1$ and $f(0) = 0$. If f is an isomorphism, then $\text{Ker}(f) = \{1\}$.**

2. Proof. (i): Let $x, y \in \text{Ker}(f)$, and $a \in L_1$ which $x \leq a$. We show that $x \wedge y \in \text{Ker}(f)$ and $a \in \text{Ker}(f)$. Since $x, y \in \text{Ker}(f)$, $f(x) = f(y) = 1$. Therefore $f(x \wedge y) = f(x) \wedge f(y) = 1 \wedge 1 = 1$ and it means that $(x \wedge y) \in \text{Ker}(f)$. If $x \leq a$, then by Definition 3.1 $a \in (a \vee x)$, so $f(a) \in f(a \vee x) = f(a) \vee f(x)$. Therefore, $f(a) \in f(a) \vee 1$. By Definition 3.1, $1 \leq f(a)$ and since 1 is the greatest element, $f(a) = 1$ and it means that $a \in \text{Ker}(f)$.

(ii): If $x \leq y$, then by Definition 3.1, $y \in x \vee y$. So $f(y) \in f(x \vee y) = f(x) \vee f(y)$ and it means that $f(x) \leq f(y)$.

(iii): Let f be an epimorphism. So, there exists $t \in L_1$ such that $f(t) = 1$. Hence $t \in \text{Ker}(f)$. Thus $\text{Ker}(f) \neq \emptyset$. Since $x \leq 1$ for all $x \in L_1$ by (ii), $f(x) \leq f(1)$, therefore $f(t) \leq f(1)$. Since $f(t) = 1$, so $1 \leq f(1)$. Thus $f(1) = 1$ (since 1 is the greatest element). Now, we prove $f(0) = 0$. Since f is onto, therefore $t \in L_1$ such that $f(t) = 0$. We have $0 \leq x$ For all $x \in L_1$, so by (ii) is implied that $f(0) \leq f(x)$, for all $x \in L_1$. Therefore $f(0) \leq f(t) = 0$ and since 0 is the least element, so $f(0) = 0$.

(iv): Let f be an isomorphism. Since every isomorphism is an epimorphism, f is an epimorphism. Thus by (iii), $f(1) = 1$. Now, we assume $x \in \text{Ker}(f)$. So $f(x) = 1 = f(1)$. Since F is also one to one, so $x = 1$. Hence $\text{Ker}(f) = \{1\}$. \square

If $f(x) \leq f(y)$, where $x \leq y$, then f is called isotone (order preserving map). If f is a good homomorphism, then by Proposition 4.18, it is an order preserving map. Distributivity (dual distributivity) is preserved by good homomorphisms.

Set $f^{-1}(F) = \{x \in L_1 | f(x) \in F\}$.

Proposition 4.19. **[(i)]**

- 1. If f is a isomorphism and F is a filter of L_1 , then $f(F)$ is a filter of L_2 .**
- 2. If F is a filter of L_2 , then $f^{-1}(F)$ is a filter of L_1 .**
- 3. Let $a \in L_1$. Then $f(F(a)) \subseteq F(f(a))$.**

Proof. (i): Let $x, y \in f(F)$ and $c \in L_2$ such that $x \leq c$. So there exist $a, b \in F$, such that $f(a) = x$ and $f(b) = y$. It is obvious that $(x \wedge y) \in f(F)$. We only,

show that $c \in f(F)$. Since f is onto, so there exists $d \in L_1$ such that $f(d) = c$. By $x \leq c$, we have $x \wedge c = x$, also $f(a \wedge d) = f(a) \wedge f(d) = f(a)$. Since f is one to one, $a \wedge d = a$. Hence $a \leq d$ and $a \in F$. Since F is a filter, so $d \in F$. Thus $f(d) \in f(F)$ that implies $c \in f(F)$.

(ii): Let $x, y \in f^{-1}(F)$ and $a \in L_1$ such that $x \leq a$. We must show that $x \wedge y \in f^{-1}(F)$ and $a \in f^{-1}(F)$. $x, y \in f^{-1}(F)$ implies $f(x), f(y) \in F$. Since F is a filter, so $f(x) \wedge f(y) \in F$. Hence $f(x \wedge y) \in F$. Therefore $x \wedge y \in f^{-1}(F)$. Since $x \leq a$ and f is a order preserving map, so $f(x) \leq f(a)$ that implies $f(a) \in F$. Thus $a \in f^{-1}(F)$.

(iii): Let $x \in f(F(a))$. Then there exists a element $b \in F(a)$ such that $x = f(b)$ and $a \leq b$. Since f is isotone, $f(a) \leq f(b)$. So $f(a) \leq x$ and it implies that $x \in x \vee f(a)$. Thus $x \in F(f(a))$. \square

Proposition 4.20. *Let f be onto good homomorphism. Then the following conditions hold:*

[(i)]If $a \in L_1$ is a distributive (dual distributive) element, then $f(a)$ is a distributive (dual distributive) element of L_2 . If L_1 is distributive (dual distributive) hyperlattice, then L_2 is so.

- 2.** *Proof.* (i): Let $x, y \in L_2$. We show that $f(a) \wedge (x \vee y) = (f(a) \wedge x) \vee (f(a) \wedge y)$. Since f is onto, so there exist $b, c \in L_1$, such that $f(b) = x$ and $f(c) = y$. Thus we have:

$$\begin{aligned} f(a) \wedge (x \vee y) &= f(a) \wedge (f(b) \vee f(c)) \\ &= f(a) \wedge (f(b \wedge c)) = f(a \wedge (b \vee c)) \\ &= f((a \wedge b) \vee (a \wedge c)) = f(a \wedge b) \vee f(a \wedge c) \\ &= [f(a) \wedge f(b)] \vee [f(a) \wedge f(c)] \\ &= (f(a) \wedge x) \vee (f(a) \wedge y). \end{aligned}$$

So $f(a)$ is a distributive element of L_2 . While a is a dual distributive element, similarly it is proved that $f(a)$ is dual distributive element of L_2 .

(ii): It is proved similar to part (i). \square

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