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IFSs consisting of generalized convex contractions

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Abstract

In this paper we introduce the concept of iterated function system consisting of generalized convex contractions. More precisely, given $n \in \mathbb{N}^*$, an iterated function system consisting of generalized convex contractions on a complete metric space (X, d) is given by a finite family of continuous functions $(f_i)_{i\in I}$, $f_i : X \to X$, having the property that for every $\omega \in \Lambda_n(I)$ there exists a family of positive numbers $(a_{\omega,v})_{v\in V_n(I)}$ such that:

α) $\max\limits_{\omega\in\Lambda_n(I)} \sum\limits_{v\in V_n(I)} a_{\omega,v} < 1$;

β) $d(f_\omega(x), f_\omega(y)) \leq \sum\limits_{v\in V_n(I)} a_{\omega,v} d(f_v(x), f_v(y))$ for all $\omega \in \Lambda_n(I)$, $x, y \in X$. Here $\Lambda_n(I)$ represents the family of words with n letters from I, $V_n(I)$ designates the family of words having at most $n-1$ letters from I, while, if $\omega = \omega_1\omega_2...\omega_p$, by f_ω we mean $f_{\omega_1} \circ f_{\omega_2} \circ ... \circ f_{\omega_p}$. Denoting such a system by $\mathcal{S} = ((X, d), n, (f_i)_{i\in I})$, one can consider the function $F_{\mathcal{S}} : \mathcal{K}(X) \to \mathcal{K}(X)$ described by $F_{\mathcal{S}}(B) = \bigcup\limits_{i\in I} f_i(B)$, for all $B \in \mathcal{K}(X)$, where $\mathcal{K}(X)$ means the set of non-empty compact subsets of X. Our main result states that $F_{\mathcal{S}}$ is a Picard operator for every iterated function system consisting of generalized convex contractions $\mathcal{S}$.

1 Introduction

As the contraction condition from Banach-Caccioppoli-Picard principle is very strong, V. Istrăţescu introduced and studied the convex contraction condition (see [9], [10] and [11]) in order to provide contraction-type conditions which do

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not imply the contraction condition but for which the existence and uniqueness of the fixed point are assured. More precisely we have the following:

Definition 1.1. *A continuous function $f : X \to X$, where (X, d) is a complete metric space, is called convex contraction if there exist $a, b \in (0, 1)$ such that $a + b < 1$ and $d(f^{[2]}(x), f^{[2]}(y)) \leq ad(f(x)), f(y)) + bd(x, y)$ for every $x, y \in X$.*

Istrăţescu proved that any convex contraction is a Picard operator. In addition he presented a convex contraction which is not contraction and introduced the following generalization of the concept of convex contraction that was also studied by S. András (see [2] and [3]):

Definition 1.2. *Given a complete metric space (X, d), a continuous function $f : X \to X$ is called a generalized convex contraction provided that there exist $n \in \mathbb{N}^*$ and $\alpha_0, \alpha_1, \ldots, \alpha_{n-1} \geq 0$ such that $\sum\limits_{i=0}^{n-1} \alpha_i < 1$ and $d(f^{[n]}(x), f^{[n]}(y)) \leq \sum\limits_{k=0}^{n-1} \alpha_k d(f^{[k]}(x), f^{[k]}(y))$ for all $x, y \in X$, where by $f^{[k]}$ we mean the composition of f by itself k times.*

They proved that each generalized convex contraction is a Picard operator. For other generalizations of Istrăţescu's result see [1], [12], [14], [16], [20] and [29].

For an iterated function system consisting of a finite family of contractions $(f_k)_{k\in\{1,2,\ldots,n\}}$, $f_k : X \to X$, where (X, d) is a complete metric space, there exists a unique non-empty compact subset A (called the attractor of the system) of X such that $A = \bigcup\limits_{k=1}^{n} f_k(A)$. This procedure gives almost all fractals and consequently several authors extended the notion of iterated function system (see [4], [5], [6], [7], [8], [13], [15], [17], [18], [19], [21], [22], [23], [25], [26], [27], [28], [30], [31], [32], [33] and [34] and the references therein). Along these lines of research R. Miculescu and A. Mihail (see [24]) introduced the concept of iterated function system consisting of convex contractions and proved the existence and uniqueness of the attractor of such a system obtaining in this way another generalization of the above mentioned Istrăţescu's result.

In this paper, combining these two directions of generalization of Istrăţescu's theorem, we study iterated function systems consisting of generalized convex contractions.

2 Preliminaries

By $f^{[n]}$ we mean the composition of the function $f : X \to X$ by itself n times.

For a family of functions $(f_i)_{i\in I}$, where $f_i : X \to X$, $\alpha_1, \alpha_2, ..., \alpha_n \in I$ and $Y \subseteq X$, we use the following notations:

– $f_{\alpha_1} \circ f_{\alpha_2} \circ ... \circ f_{\alpha_n} \overset{not}{=} f_{\alpha_1\alpha_2....\alpha_n}$

– $f_{\alpha_1\alpha_2....\alpha_n}(Y) \overset{not}{=} Y_{\alpha_1\alpha_2....\alpha_n}$.

For a metric space (X, d), by $\mathcal{K}(X)$ we mean the set of non-empty compact subsets of X.

B^A represents the set of functions from A to B.

For a set I, we use the following notations:

– $I^{\mathbb{N}^*} \overset{not}{=} \Lambda(I)$; hence the elements of $\Lambda(I)$ can be written as infinite words $\omega = \omega_1\omega_2...\omega_n...$ with letters from I

– $I^{\{1,2,...,n\}} \overset{not}{=} \Lambda_n(I)$; hence the elements of $\Lambda_n(I)$ can be written as words $\omega = \omega_1\omega_2...\omega_n$ with n letters from I

– $\Lambda_0(I) \cup \Lambda_1(I) \cup ... \cup \Lambda_{n-1}(I) \overset{not}{=} V_n(I)$, where $\Lambda_0(I) = \{\lambda\}$ is the set consisting on the empty word; hence $V_n(I)$ is the set of all words having at most $n-1$ letters from I

– $\underset{n\in\mathbb{N}}{\cup} \Lambda_n(I) \overset{not}{=} \Lambda^*(I)$; hence $\Lambda^*(I)$ is the set of all finite words with letters from I.

For a function $f : X \to X$, by f_λ we mean Id_X. For $\alpha \in \Lambda(I) \cup \Lambda_n(I)$ and $m \leq n$, we use the following notation: $\alpha_1\alpha_2....\alpha_m \overset{not}{=} [\alpha]_m$.

By $\alpha\beta$ we understand the concatenation of the words $\alpha \in \Lambda^*(I)$ and $\beta \in \Lambda(I) \cup \Lambda^*(I)$.

Definition 2.1. *For a metric space (X, d), we consider the Hausdorff-Pompeiu metric $h : \mathcal{K}(X) \times \mathcal{K}(X) \to [0, +\infty)$ described by*

$$h(A, B) = \max\{\sup_{x\in A}(\inf_{y\in B} d(x, y)), \sup_{x\in B}(\inf_{y\in A} d(x, y))\},$$

for every $A, B \in \mathcal{K}(X)$.

Proposition 2.2. (see Proposition 2.1 from [26]) *For a metric space (X, d), we have*

$$h(\underset{i\in I}{\cup} H_i, \underset{i\in I}{\cup} K_i) \leq \sup_{i\in I} h(H_i, K_i),$$

for every $(H_i)_{i\in I}$ and $(K_i)_{i\in I}$ finite families of elements from $\mathcal{K}(X)$.

Proposition 2.3. (see Theorem 2.1 from [26]) *If the metric space (X, d) is complete, then $(\mathcal{K}(X), h)$ is a complete metric space.*

Definition 2.4. *For a metric space (X, d), we consider the function $\delta : \mathcal{K}(X) \times \mathcal{K}(X) \to [0, +\infty)$ defined by*

$$\delta(A, B) = \sup_{x \in A, y \in B} d(x, y),$$

for all $A, B \in \mathcal{K}(X)$.

Remark 2.5. *For all $A, B \in \mathcal{K}(X)$ we have*

$$h(A, B) \leq \delta(A, B).$$

Definition 2.6. *A function $f : X \to X$, where (X, d) is a metric space, is called Picard operator if there exists a unique $x^* \in X$ such that $f(x^*) = x^*$ and $\lim_{k \to \infty} f^{[k]}(x) = x^*$ for every $x \in X$.*

3 The main result

Definition 3.1. *Given $n \in \mathbb{N}^*$, an iterated function system consisting of generalized convex contractions on a complete metric space (X, d) is given by a finite family of continuous functions $(f_i)_{i \in I}$, $f_i : X \to X$, such that for every $\omega \in \Lambda_n(I)$ there exists a family of positive numbers $(a_{\omega,v})_{v \in V_n(I)}$ such that:*

α)

$$\max_{\omega \in \Lambda_n(I)} \sum_{v \in V_n(I)} a_{\omega,v} < 1;$$

β)

$$d(f_\omega(x), f_\omega(y)) \leq \sum_{v \in V_n(I)} a_{\omega,v} d(f_v(x), f_v(y)),$$

for all $\omega \in \Lambda_n(I)$, $x, y \in X$.

We denote such a system by

$$\mathcal{S} = ((X, d), n, (f_i)_{i \in I}).$$

One can consider the function $F_{\mathcal{S}} : \mathcal{K}(X) \to \mathcal{K}(X)$ described by

$$F_{\mathcal{S}}(B) = \bigcup_{i \in I} f_i(B)$$

for all $B \in \mathcal{K}(X)$.

Remark 3.2. *If the set I has one element, $((X,d),n,(f_i)_{i\in I})$ is nothing else but the notion of generalized convex contraction. Note also that the notion of iterated function system consisting of convex contractions from [24] is a particular case of the one of iterated function system consisting of generalized convex contractions (just take $n=2$).*

Theorem 3.3. *$F_{\mathcal{S}}$ is a Picard operator, for every iterated function system consisting of generalized convex contractions $\mathcal{S}$.*

Proof. In the sequel we shall use the following notations:

– $\mathcal{S}=((X,d),n,(f_i)_{i\in I})$

– $\sum\limits_{v\in V_n(I)} a_{\omega,v} \overset{not}{=} d_\omega$

– $\max\limits_{\omega\in\Lambda_n(I)} d_\omega \overset{not}{=} d<1$

– $y_k(Y,Z) = \max\{x_{k-n+1}(Y,Z), x_{k-n+2}(Y,Z), ..., x_k(Y,Z)\}$, where $x_k(Y,Z)=\sup\limits_{\omega\in\Lambda_k(I)} \delta(f_\omega(Y),f_\omega(Z))$, $Y,Z\in\mathcal{K}(X)$, $k\in\mathbb{N}^*$, $k\geq n-1$ and $Y,Z\in\mathcal{K}(X)$.

When no confusion is possible, we denote $y_k(Y,Z)$ by y_k and $x_k(Y,Z)$ by x_k.

Claim 3.4. *The sequence $(y_{k+n}(Y,Z))_{k\in\mathbb{N}^*}$ is decreasing for all $Y,Z\in\mathcal{K}(X)$.*

Justification of Claim 3.4. Given $u\in\Lambda_{k+n}(I)$ there exist $\omega\in\Lambda_n(I)$ and $q\in\Lambda_k(I)$ such that $u=\omega q$ and therefore we have

$$d(f_u(y),f_u(z)) = d(f_{\omega q}(y),f_{\omega q}(z)) \leq$$

$$\leq \sum_{v\in V_n} a_{\omega,v} d(f_{vq}(x),f_{vq}(y)) \leq a_\omega x_k + \sum_{v\in\Lambda_1(I)} a_{\omega,v}x_{k+1} + ... + \sum_{v\in\Lambda_{n-1}(I)} a_{\omega,v}x_{k+n-1} \leq$$

$$\leq \max\{x_k,x_{k+1},...,x_{k+n-1}\}(a_\omega + \sum_{v\in\Lambda_1(I)} a_{\omega,v} + ... + \sum_{v\in\Lambda_{n-1}(I)} a_{\omega,v}) \leq$$

$$\leq y_{k+n-1}\sum_{v\in V_n(I)} a_{\omega,v} = d_\omega y_{k+n-1} \leq d y_{k+n-1},$$

for all $y\in Y$ and $z\in Z$, so, by passing to supremum over $y\in Y$ and $z\in Z$, we deduce that $\delta(f_u(Y),f_u(Z)) \leq dy_{k+n-1}$ for all $k\in\mathbb{N}^*$. By passing to supremum over $u\in\Lambda_{k+n}(I)$, we get that

$$x_{k+n} = \sup_{\omega\in\Lambda_{k+n}(I)} \delta(f_\omega(Y),f_\omega(Z)) \leq dy_{k+n-1} < y_{k+n-1}, \qquad (*)$$

for all $k \in \mathbb{N}^*$. As

$$x_{k+1}, x_{k+2}, ..., x_{k+n-1} \leq \max\{x_k, x_{k+1}, ..., x_{k+n-1}\} = y_{k+n-1}, \qquad (**)$$

from $(*)$ and $(**)$, we infer that $y_{k+n} = \max\{x_{k+1}, x_{k+2}, ..., x_{k+n}\} \leq y_{k+n-1}$ for all $k \in \mathbb{N}^*$ and the justification of Claim 3.4 is done.

Claim 3.5. $\lim\limits_{k\to\infty} x_k(Y,Z) = 0$ *for all* $Y, Z \in \mathcal{K}(X)$.

Justification of Claim 3.5 Based on $(*)$ from the proof of Claim 3.4, we have

$$y_{k+2n-1} = \max\{x_{k+n}, x_{k+1+n}, ..., x_{k+2n-1}\} \leq$$
$$\leq \max\{dy_{k+n-1}, dy_{k+n}, ..., dy_{k+2n-2}\},$$

so, taking into account Claim 3.4, we get that $y_{k+2n-1} \leq dy_{k+n-1}$ for all $k \in \mathbb{N}^*$. Consequently $y_{ni} \leq d^{i-1}y_n$, $y_{ni+1} \leq d^{i-1}y_{n+1} \leq d^{i-1}y_n$, ..., $y_{ni+n-1} \leq d^{i-1}y_{2n-1} \leq d^{i-1}y_n$ for all $i \in \mathbb{N}^*$, hence the series $\sum\limits_{k=n}^{\infty} y_k$ is convergent. Using $(*)$ from the proof of Claim 3.4 and the comparison test, we conclude that the series $\sum\limits_{k=n+1}^{\infty} x_k$ is convergent and, consequently, $\lim\limits_{k\to\infty} x_k = 0$. The justification of Claim 3.5 is done.

Claim 3.6. *For every* $Y \in \mathcal{K}(X)$, *the sequence* $(F_{\mathcal{S}}^{[k]}(Y))_{k\in\mathbb{N}^*}$ *is convergent.*

Justification of Claim 3.6 Since

$$h(F_{\mathcal{S}}^{[k]}(Y), F_{\mathcal{S}}^{[k]}(Z)) = h(\bigcup_{\omega\in\Lambda_k(I)} f_\omega(Y), \bigcup_{\omega\in\Lambda_k(I)} f_\omega(Z)) \overset{\text{Proposition 2.2}}{\leq}$$

$$\leq \sup_{\omega\in\Lambda_k(I)} h(f_\omega(Y), f_\omega(Z)) \overset{\text{Remark 2.5}}{\leq} x_k, \qquad (*)$$

for every $Y, Z \in \mathcal{K}(X)$, $k \in \mathbb{N}^*$, we deduce that

$$\lim_{k\to\infty} h(F_{\mathcal{S}}^{[k]}(Y), F_{\mathcal{S}}^{[k]}(Z)) = 0. \qquad (1)$$

Choosing $Z = F_{\mathcal{S}}(Y)$, from $(*)$, the convergence of the series $\sum\limits_k x_k$ and the comparison test, we infer that the series $\sum\limits_{k\in\mathbb{N}^*} h(F_{\mathcal{S}}^{[k+1]}(Y), F_{\mathcal{S}}^{[k]}(Y))$ is convergent for all $Y \in \mathcal{K}(X)$. Therefore $(F_{\mathcal{S}}^{[k]}(Y))_{k\in\mathbb{N}^*}$ is a Cauchy sequence and, because $(\mathcal{K}(X), h)$ is complete (see Proposition 2.3), it is convergent. The justification of Claim 3.6 is done.

Claim 3.6 assures us that if $Y, Z \in \mathcal{K}(X)$, then there exist $A_Y, A_Z \in \mathcal{K}(X)$ such that

$$\lim_{k\to\infty} h(F_{\mathcal{S}}^{[k]}(Y), A_Y) = 0 \text{ and } \lim_{k\to\infty} h(F_{\mathcal{S}}^{[k]}(Z), A_Z) = 0. \tag{2}$$

Since $h(A_Y, A_Z) \leq h(A_Y, F_{\mathcal{S}}^{[k]}(Y)) + h(F_{\mathcal{S}}^{[k]}(Y), F_{\mathcal{S}}^{[k]}(Z)) + h(F_{\mathcal{S}}^{[k]}(Z), A_Z)$ for all $k \in \mathbb{N}^*$, from (1) and (2) we obtain that $A_Y = A_Z \overset{def}{=} A$ for every $Y, Z \in \mathcal{K}(X)$. Hence $\lim\limits_{k\to\infty} h(F_{\mathcal{S}}^{[k]}(B), A) = 0$ for every $B \in \mathcal{K}(X)$. The remaining part of the proof goes as in Theorem 3.2 from [24] and, consequently, we just mark the main steps:

a) For each $\omega \in \Lambda(I)$ there exists $a_\omega \in X$ such that

$$\lim_{k\to\infty} \sup_{\omega\in\Lambda(I)} h(f_{[\omega]_k}(B), \{a_\omega\}) = 0,$$

for every $B \in \mathcal{K}(X)$.

b) $$A = \overline{\{a_\omega \mid \omega \in \Lambda(I)\}}.$$

c) The implication

$$\lim_{k\to\infty} h(Y_k, Y) = 0 \Rightarrow \lim_{k\to\infty} h(F_{\mathcal{S}}(Y_k), F_{\mathcal{S}}(Y)) = 0$$

is true for every $(Y_k)_{k\in\mathbb{N}} \subseteq \mathcal{K}(X)$ and $Y \in \mathcal{K}(X)$.

d) A is the unique fixed point of $F_{\mathcal{S}}$. $\square$

Conclusions. In this paper we consider a special type of iterated function systems, namely those consisting of generalized contractions. We proved that such a system has a unique attractor, obtaining in this way a generalization of a fixed point theorem concerning generalized convex contractions which is due to V. Istrăţescu. In a future paper we intend to study iterated function systems consisting of generalized convex contractions in the more general setting of b-metric spaces.

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